Name: $\qquad$

Task Master: $\qquad$ Cynic: $\qquad$ Recorder: $\qquad$

## CLIMBING A HILL

Working in small groups (3 or 4 people), solve as many of the problems below as possible. Try to resolve questions within the group before asking for help. Each group member should then write up the solutions in their own words; Show your work! Full credit will only be given if your answer is supported by calculations and/or explanations as appropriate.

You and a friend decide to climb a hill, taking separate paths. You have a topographic map, which uses rectangular coordinates $(x, y)$ measured in miles. Your global positioning system says that your friend is at point $\mathbf{A}$ or $\mathbf{B}$ (pick one), and that you are at one of the remaining points (pick one):
A: $(2,0)$
B: $(0,3)$
C: $(1,4)$
D: $(1,-4)$
E: $(-4,9) \quad$ F: $(4,-9)$

Your guidebook has a map, which is reproduced below, and which shows contour lines every 100 feet. The guidebook also tells you that the height $h$ of the hill in feet above sea level is given by

$$
h=5000-a x^{2}-b y^{2}
$$

where $a=30 \frac{\mathrm{ft}}{\mathrm{mi}^{2}}$, and $b=10 \frac{\mathrm{ft}}{\mathrm{mi}^{2}}$.

1. Starting at your present locations, which way should each of you go in order to climb the hill as steeply as possible? Draw these vectors on your topographic map.
2. How steep is the hill at each of these points if you go in this direction?
3. Draw a contour diagram of the hill very close to your location, that is, draw the contour diagram of the tangent plane there.
4. How steep is the hill at your location if you go due east (the $x$-direction)? Northeast?

