Dare to Compare?

Introduction to Using Comparison Charts in the Mathematics Classroom

Dave Damcke (*University of Portland)*, Tevian Dray (*Oregon State University*), Maria Fung (*Western Oregon University*), Dianne Hart (*Oregon State University*), Lyn Riverstone (*Oregon State University*)

The geometry team at the Oregon Mathematics Leadership Institute (OMLI) created and delivered a course for K-12 teachers called “Comparing Different Geometries.” A major part of this course was spent on creating extensive comparison charts aimed at synthesizing newly acquired knowledge that teachers gained in Spherical and Taxicab geometries, and connecting this knowledge to ideas in the school geometry curriculum.

In this article, we discuss using comparison as a method for reviewing, looking back, and extending mathematical knowledge. We can also imagine it working well as part of an on-going process; as students construct new knowledge, they may spend time revising and editing their charts based on formative feedback from the teacher. We believe that comparison charts are an interesting and invaluable tool for supporting students as they make connections between different mathematical ideas (NCTM, 2000).

What to Compare?

Mathematics is a subject rich in interrelations and surprising analogies. There are numerous possibilities for comparing different objects, operations, or properties. We will suggest several very different examples to illustrate how comparison can be used in the mathematics classroom.

- In geometry, students often struggle with definitions. Remembering which properties define special types of quadrilaterals can be tricky for many students. Using a comparison chart where students compare, for instance, squares with rectangles, rhombi with parallelograms, and trapezoids with parallelograms, could be a great way to remember the characteristic properties of these different polygons. Students could be given the task of describing in detail the similarities and differences between each pair. Once this activity is completed and thoroughly discussed, students could also be given the task to compare and contrast pairs of different types of triangles (e.g. isosceles and right triangles, scalene and right triangles, etc.).

  Another geometric topic students could be asked to focus on is circles. What properties do all circles share? Students may consider that π is the ratio of the circumference to the diameter of any circle, for example. Are there properties that depend on the specific circle we are considering? By looking for ways to compare circles in this way, students would have the opportunity to look back at the concepts of radius, circumference, and area, to discuss their definitions and to consider examples.

- When teaching the number and operation strand in upper elementary and middle school, comparing the commutative, associate, and distributive properties, the existence of additive and multiplicative identities and inverses, and the closure properties for the operations of addition and multiplication can help students master some of these basic properties of operations with whole numbers, integers, and rational numbers. For example, students might compare the commonalities and differences as one expands the set of the whole numbers to the integers, yielding numbers closed under additive inverses, and the integers to the rational numbers yielding numbers closed under multiplicative inverses.

- In statistics students are introduced to a variety of graphs and charts as a way of organizing data. They often struggle to make sense of which properties of data are best captured by dot plots, histograms, box-and-whiskers plots, scatter plots and pie charts. A comparison chart could be constructed where students are asked to specify, compare and contrast the utility properties of each type of graph as well as the similarities among them.

- In other classes, students could be asked to make comparison charts between two or more given graphs; for instance, one that displays distance traveled versus time and one showing speed versus time. What do these two graphs have in common? How are they related? What makes them different?

- In trigonometry, students might be asked to compare a sine function to a tangent function. A part of their task could be to identify the properties they share and those they differ by. In algebra, a similar task can be assigned for polynomial functions of even and odd degrees.

How to Compare?

In our geometry class we tried two different approaches for generating the rows of a comparison chart. In our first approach, we asked our class to come up with the different areas for comparison. This was done by first giving the students sufficient private think time, then having them share in a small group go-around, and finally following this up with a whole group discussion where the instructor records all the ideas on the board or on a big poster for all to see. The pedagogical advantage of this approach is that the teacher could focus on the areas she sees as most important for comparison, while still allowing students ownership of their ideas. In our second approach, the teacher provides the class with the areas of comparison. This might be the most efficient method for focusing the comparison process, but it does not allow students to discover by themselves the important facets to be considered.

Here are a couple examples of partial comparison charts:

Squares and Rectangles	Similarities	Differences
Properties to be compared are generated by the class: Sides Angles	Both have 4 sides and 4 right angles. Both have 2 pairs of parallel sides.	In squares all 4 sides are always congruent. This property is not necessary for rectangles.

Isosceles and right triangles (Here the teacher provides the students with the following chart headings.)	Similarities	Differences
Angles	Both kinds of triangles have the sum of the measures of the three angles equal to 180 degrees. There can be right isosceles triangles, where the three angles measure 45, 45, and 90 degrees.	An isosceles triangle can be equilateral, but an equilateral triangle can never be right. In right triangles, there is always exactly one right angle; in isosceles triangles this is not necessary.
Sides	Three sides.	In isosceles triangles, two of the sides are always congruent; in a right triangle, this is not necessary but it is possible. Right triangles can be scalene (e.g. 3-4-5 sided right triangle). The Pythagorean Theorem holds for right triangles but not for non-right isosceles triangles.

Whichever method a teacher chooses for doing comparison, we believe constructing comparison charts is an excellent example of a group-worthy task. A teacher could decide on the membership of each group, given the specifics of the class and students she has. We suggest the use of group roles within each group in order to make sure that everyone is taking part in the comparison activity. One possibility for assigning group roles goes as follows: group captain, explainer 1, explainer 2, and recorder. The group captain's main responsibility is to keep the group on task by allowing for private thinking time first, and then orchestrating a go-around protocol with each member of the group taking turns sharing ideas. The captain is also in charge of ensuring that the entire task is completed by the group. The two explainers (you could just use one in groups of three students) have the responsibility of providing justification for each line in the comparison charts. The recorder is to keep a written record of the entire group's contribution. It is probably best, in terms of accountability, if students are warned in advance that a random person from each group will report out to the entire class. It is crucial that the teacher observes and takes notes of each group's contribution, so she can sequence the presentations of ideas so that conceptual problems or mistakes can be fixed by the class and the main mathematical ideas are fleshed out.

Oftentimes, comparison charts can be a great vehicle for doing a jigsaw-puzzle group-work protocol. A teacher could give each of her groups a separate part of the comparison chart and instruct them in becoming experts on their assigned topic. For example, one group could become experts in comparing trapezoids with parallelograms, and another in comparing squares and rectangles. Then new groups could be formed with members that are experts on different topics, and new ideas can be learned by sharing in these new groups first, and then reporting back to the original groups. An alternative to this scenario can involve the expert group making a poster with their findings and then either sharing it in front of the rest of the students, or doing a walk through the classroom, with the different posters hanging on the wall, where feedback in the form of questioning is provided by the rest of the groups.

It is important to turn the creation of comparison charts into a fun-filled event where students work together and learn together. We believe this relaxed and reflective atmosphere is conducive to learning for all students.

When to Compare?

It seems that the best times to use comparison charts seem to be at the end of a unit, as a review for a test, or simply any time students seem ready to gain a new perspective or make connections between ideas.

It is not necessary to do a complete comparison—even partial results can go a long way toward creating reflection opportunities for students and teachers alike.

It is our sincere hope that more teachers decide to use this wonderful teaching tool. So, do you dare to compare?

References:

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.