Showalter, R. E.; Ning Su Partially Saturated Flow in a Composite Poroelastic Medium,
Poromechanics II, (Grenoble, 2002), J.-L. Auriault et al (editors), pp.549--554, Balkema, Lisse, 2002.
TICAM Report 02-15.

PARTIALLY SATURATED FLOW IN A COMPOSITE POROELASTIC MEDIUM

R.E. SHOWALTER AND NING SU

Abstract. (Preliminary Report.) The model formulation and existence theory is described for diffusion of a barotropic fluid through a partially saturated poroelastic composite medium consisting of two components. This includes the Barenblatt-Biot double-diffusion model of elastic deformation and laminar flow in a fissured medium, such as consolidation processes in a system of fissures distributed throughout a matrix of highly porous cells. Nonlinear effects of density, saturation, porosity and permeability variations with pressure are included, and the seepage surfaces are determined by variational inequalities on the boundary.

1. INTRODUCTION

The classical linear model of transient flow and deformation of a homogeneous fully-saturated elastic porous medium depends on an appropriate coupling of the fluid pressure and solid stress. The total stress consists of both the *effective stress* given by the strain of the structure and the *pressure* arising from the pore fluid. The local storage of fluid mass results from increments in the density of the fluid and the dilation of the structure. The combinations of the fluid mass conservation with Darcy's law for laminar flow and of the momentum balance equations with Hooke's law for elastic deformation result in the *Biot diffusion-deformation model* of poroelasticity. Its application to consolidation requires the *quasi-static* modification in which the dynamic momentum equations are replaced by the corresponding equilibrium equations. See [12], [13], [21], [3], [18], [34].

The description of flow in a rigid fully–saturated but heterogeneous medium often requires several distinct spatial scales for porosity and permeability. The simplest and most frequently used model which allows for qualitatively different properties is the *Barenblatt double–diffusion model*, which consists of the combined effects of two distinct components in parallel. Both of these components occur locally in any representative volume element, and they behave as two independent diffusion processes which are coupled by a distributed exchange term that, in its simplest form, is proportional to the difference in pressure between fluids in the two components. In the special case which is used to model naturally fractured media, the first component of the model is the highly developed fracture system and the second is the porous matrix structure. See [6], [4], [8], [30].

Date: April, 2002.

2000 *Mathematics Subject Classification.* Primary 35K50, 35B25; Secondary 80A20, 35F15.

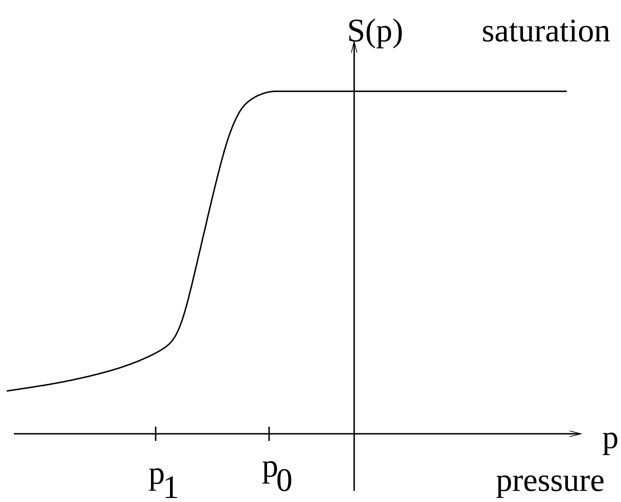
This material is based in part upon work supported by the Texas Advanced Research Program.

The research of the second author was supported by the Foundation for University Key Teacher by the Ministry of Education of China and by a Visiting Faculty Fellowship from the Texas Institute for Computational and Applied Mathematics.

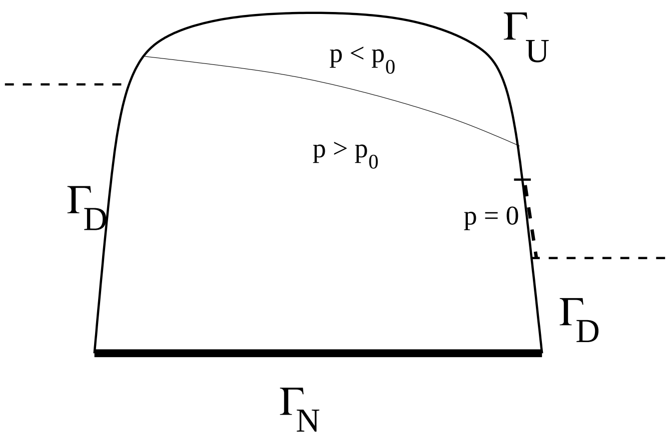
The basic ideas of poroelasticity continue to play an important role in the more complex models of double–diffusion combined with deformation. Since both of the pressure fields contribute to the stress field of the structure, it is necessary to incorporate Biot's concepts into the Barenblatt model. The momentum equations contain contributions to total stress from each of the two pressure fields, and the two equations of fluid transport follow from the continuity of fluid mass and consideration of the effects of *dilation* of the structure on the flow in both of the components. The fluid transport within this composite deformable porous medium is described by a pair of pressure equations for diffusion in the respective components of the medium together with an exchange term. This simplistic combination of the Barenblatt double–diffusion model with the Biot diffusion–deformation model has been developed and used extensively in the engineering literature. See [10], [11], [31], [18], [28], [9], [19], [25].

Mathematical issues of model development and the theory of well–posedness of the initial-boundary-value problem for a homogeneous fully–saturated *elastic* porous medium were first studied in the fundamental work of [3]. They derived a non-isotropic form of the Biot diffusion–deformation system by homogenization and then obtained a *strong* solution. In the later paper of [32] the *weak* solution is obtained in the first order Sobolev space $H^1(\Omega)$. The existence, uniqueness, and regularity theory for the Biot system together with extensions to include the possibility of viscous terms arising from secondary consolidation and the introduction of appropriate boundary conditions at both closed and drained interfaces were given in [24]. These results were extended to the *Barenblatt–Biot double–diffusion deformation model* in [25].

In all of the preceding works, it was assumed that the medium is fully–saturated. By *partially–saturated* we mean here that the fluid saturation level is given by a continuous monotone function of pressure which increases from near zero to unity in the vicinity of the (negative) *capillary tension* or *air-entry pressure value*, p_0. The air is assumed to remain at a constant atmospheric pressure, so this model does not account for the effects of increments in air pressure, such as occur in regions of fluid-entrapped air.



The permeability is likewise pressure dependent, and these relationships are available from experiment. See [7], especially Section 9.4, for background information on partially–saturated, *i.e.*, unsaturated flow. A mathematical theory was developed in the fundamental work of [5] and [27] for a special limiting case in which the saturation is given by the Heaviside step function of pressure. The usual situation with a continuous saturation was developed by [15], [29], [17], [1] and [2]. The boundary of of the region Ω is given by the disjoint union of the parts Γ_D where pressure is specified and Γ_{fl}. The flux boundary Γ_{fl} is further written as the disjoint union of Γ_N where flux q is prescribed and Γ_U which is exposed to the air. Here the fluid pressure p cannot exceed the outside null pressure of air, and there can be no flow into Ω. Also, $p = 0$ on the *seepage surface*, which is that part of Γ_U where flux $q > 0$, and there is no flow from the boundary above that, where $p < 0$. These unilateral conditions are characterized by a *variational inequality* on Γ_U. All of these results are for the case of a *rigid homogeneous medium*.



The extension of this theory to cover both elastic deformation and partial–saturation in a homogeneous medium was given recently in [26]. This was the first mathematical proof of existence to include both aspects. See [16] for a careful discussion of modeling issues for partially saturated flow in a deformable medium. Also see [34], [33], [20], and [22] for additional perspectives in modeling and numerical simulation.

2. THE MODEL

Next we shall describe a system modeling double-diffusion in a *partially–saturated* elastic porous medium $\Omega \subset \mathbb{R}^3$. The functions $p_1(x,t)$ and $p_2(x,t)$ are the two component *pressures* at the point $x \in \Omega$ obtained by averaging the fluid pressure in the respective components over a small representative neighborhood that contains parts of both components. These are the basic variables and are described in detail below.

The fluid is assumed to be *barotropic*, *i.e.*, the density and pressure are related by a *state equation* $\rho = \rho(p)$ in which the non-decreasing constitutive function $\rho(\cdot)$ characterizes the type of fluid. For $j = 1, 2$, the function $\varphi_j(\cdot)$ is *porosity*, $S_j(\cdot)$ is the *saturation* level, and $k_j(\cdot)$ is the *relative permeability* for the laminar flow in the j-th component of the medium. Each of these functions is non-negative and pressure dependent. Let the negative pressure $\tau_j \leq 0$ denote the *capillary tension* or the *air entry pressure* in the j-th component; for a fissured medium consisting of fractures ($j = 1$) and blocks ($j = 2$), we have $\tau_2 \ll \tau_1 \leq 0$.

Each saturation function $S_j(\cdot)$ is monotone with $S_j(p) = 1$ for $p \geq \tau_j$, and $0 \leq S_j(p) < 1$ for $p \leq \tau_j$. In the context of soil mechanics, the fracture component of the medium is fully–saturated in the *groundwater region*, $\{x \in \Omega : \ p_1(x,t) > \tau_1\}$, while in the *capillary fringe*, $\{x \in \Omega : \ p_1(x,t) < \tau_1\}$, it is only partially–saturated. There is a similar pair of regions associated with the blocks. The *phreatic surfaces* $\{x \in \Omega : \ p_j(x,t) = \tau_j\}$ are unknown interfaces that separate each component into these regions.

The (small) *displacement* of the structure from the position $x \in \Omega$ is denoted by $\mathbf{u}(x,t)$. The (linearized) *strain* tensor $\varepsilon_{kl}(\mathbf{u}) \equiv \frac{1}{2}(\partial_k u_l + \partial_l u_k)$ provides a measure of the local deformation of the body. The *total stress* σ_{ij} is the sum of the *effective stress* of the purely elastic *isotropic* structure given by *Hooke's law*, $\sigma'_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij}$, with positive Lamé constants λ for dilation and μ for shear, and of the *effective pressure* stress of the fluid on the structure, hence,

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij}(\alpha_1 \chi_1(p_1)\, p_1 + \alpha_2 \chi_2(p_2)\, p_2)\,.$$

The constants α_1 and α_2 measure changes of porosity and *fluid content* of the respective components due to a volume dilation $\nabla \cdot \mathbf{u} = \varepsilon_{kk}(\mathbf{u})$ of the structure. The *Bishop parameter* $\chi_j(\cdot)$ is the fraction of pore surface of the j-th component in contact with the fluid. Little is known quantitatively about the Bishop parameter, except that it is well approximated in many situations by $\chi_j(p) \approx S_j(p)$. See [16] for further discussion.

The double-diffusion-deformation system consists of the *equilibrium equation* for momentum conservation and the two *storage equations* for fluid mass conservation. If we neglect gravity, it takes the form

$$-(\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) - \mu\Delta\mathbf{u} + \nabla\big(\alpha_1\chi_1(p_1)\,p_1 + \alpha_2\chi_2(p_2)\,p_2\big) = \mathbf{f}_0(x,t)\,, \tag{1a}$$

$$\frac{\partial}{\partial t}\big(\varphi_1(p_1)S_1(p_1)\rho(p_1) + \alpha_1\nabla\cdot\mathbf{u}\big) - \nabla\cdot\big(\rho(p_1)\,k_1(p_1)\nabla p_1\big) + \Gamma(p_1,p_2) = g_1(x,t)\,, \tag{1b}$$

$$\frac{\partial}{\partial t}\big(\varphi_2(p_2)S_2(p_2)\rho(p_2) + \alpha_2\nabla\cdot\mathbf{u}\big) - \nabla\cdot\big(\rho(p_2)\,k_2(p_2)\nabla p_2\big) - \Gamma(p_1,p_2) = g_2(x,t)\,, \tag{1c}$$

with *Darcy's law* for the filtration velocity in the respective components and a related *exchange rate* $\Gamma(p_1,p_2)$ for pressure-driven fluid transfer between the components. These are described below.

2.1. **The Exchange Term.** The two pressures $p_1(x,t)$ and $p_2(x,t)$ in the respective components are obtained by spatially–averaging over the corresponding components within a representative neighborhood centered at x. In order to describe them together with the corresponding exchange term in (1), we consider a *small* neighborhood $\Omega^\varepsilon(x_0)$ of size $\varepsilon > 0$ at a fixed point $x_0 \in \Omega$. In that neighborhood, we see three parts, namely, Ω_1^ε and Ω_2^ε which represent the two *flow regions* and the third part Ω_3^ε which is the *transition region* which separates these two flow regions. The geometric interface between Ω_3^ε and Ω_j^ε is denoted by $\Gamma_{j3}^\varepsilon \equiv \partial\Omega_3^\varepsilon \cap \partial\Omega_j^\varepsilon$ for $j = 1,2$, and $\Gamma_{33}^\varepsilon \equiv \partial\Omega_3^\varepsilon \cap \partial\Omega^\varepsilon(x_0)$ will denote the remaining boundary of Ω_3^ε. The pressure function $p(x,t)$ is defined and continuous throughout the neighborhood. In the j-th component of the medium, $j = 1,2$, the corresponding flow rate in Darcy's law is determined by the *fluid conductivity* $\rho(p)k_j(p)$. Denote by $K_j(p) \equiv \int_0^p \rho(s)k_j(s)\,ds$ the *flow potential* corresponding to the *mass flux* $= -\nabla K_j(p) = -\rho(p)k_j(p)\nabla p$ within the j-th component. Let $K(\cdot)$ be a flow potential in Ω_3^ε, and assume that $K(p)$ is a continuously differentiable connection to each $K_j(p)$

across Γ^ε_{j3}. These interface conditions correspond to continuity of pressure and fluid mass flux. The pressure is expected to be nearly constant in each of Ω^ε_1 and Ω^ε_2, and these values can be essentially different, so the primary variations in pressure will occur within Ω^ε_3. Thus, we shall assume that in the transition region Ω^ε_3 the spatial gradients are large of order $O(\frac{1}{\varepsilon})$. In order to quantify the sizes, we make a change of scale $x = \varepsilon y$ to get corresponding regions Ω_j for $j = 1, 2, 3$ of unit size in which we have

$$\begin{aligned}
a_1(p)_t &= \frac{1}{\varepsilon^2}\Delta_y K_1(p) + f_1(\varepsilon y, t), \quad y \in \Omega_1\,,\\
a_2(p)_t &= \frac{1}{\varepsilon^2}\Delta_y K_2(p) + f_2(\varepsilon y, t), \quad y \in \Omega_2\,,\\
\varepsilon^2 a_3(p)_t &= \Delta_y K(p) + \varepsilon^2 f_3(\varepsilon y, t), \quad y \in \Omega_3\,,
\end{aligned}$$

with corresponding interface conditions as before. The coefficients have been arranged to indicate that gradients in Ω_1 and Ω_2 are small of order $O(\varepsilon)$. All the ε dependence has been moved to the coefficients!

Now we can go to the limit as $\varepsilon \to 0$. In the limit, we get $\nabla_y K_j(p(y,t)) = 0$ in Ω_j, so each $K_j(p_j(x_0,t)) = K(p(y,t))$ on the rescaled interface Γ_{j3} for $j = 1, 2$. In particular, the function $u(y,t) \equiv K(p(y,t))$ satisfies the *local boundary-value problem*

$$\begin{aligned}
\Delta_y u(y,t) &= 0 \ \text{ in } \Omega_3\,,\\
u(y,t) &= K_j(p_j(x_0,t)) \ \text{ on } \Gamma_{j3} \ \text{ for } j = 1, 2,\\
\frac{\partial u}{\partial n} &= \nabla_y u \cdot \mathbf{n} = 0 \ \text{ on } \Gamma_{33}\,.
\end{aligned}$$

We have used $\mathbf{n}$ to denote the *unit outward normal* on $\partial\Omega_3$. The last condition means that we have isolated the cell in order to restrict the response to local contributions to the flux. Since this problem is linear, we can represent its solution in terms of the *cell problem*

$$\begin{aligned}
\Delta_y U(y) &= 0 \ \text{ in } \Omega_3\,, & U(y) &= 1 \ \text{ on } \Gamma_{13},\\
U(y) &= 0 \ \text{ on } \Gamma_{23}, & \frac{\partial U}{\partial n} &= 0 \ \text{ on } \Gamma_{33},
\end{aligned}$$

which defines the *characteristic flow potential* $U(\cdot)$ in Ω_3. Note that $1 - U(y)$ is the solution of the corresponding problem with 1 and 0 interchanged in the boundary conditions on Γ_{j3}. Since $K_j(p_j)$ is independent of y, we can use the cell problem to represent the solution of the local boundary-value problem as

$$u(y,t) = K_1(p_1(x_0,t))U(y) + K_2(p_2(x_0,t))(1 - U(y))\,.$$

Finally, we compute the flux q_{j3} across Γ_{j3} into Ω_3 for $j = 1, 2$, by

$$q_{13} = \int_{\Gamma_{13}} \frac{\partial u}{\partial n}\,ds = -q_{23} = \int_{\Gamma_{13}} \frac{\partial U}{\partial n}\,ds\big(K_1(p_1(x_0,t)) - K_2(p_2(x_0,t))\big)$$

These give the exchange term in the form

$$\Gamma(p_1,p_2)(x_0,t) = \kappa(x_0)\big(K_1(p_1(x_0,t)) - K_2(p_2(x_0,t))\big), \tag{1d}$$

and this completes the Barenblatt-Biot system (1). Note that all the effects of the micro-structure geometry are contained in the characteristic flow potential $U(\cdot)$. Moreover, by the maximum principle, $\partial U/\partial n > 0$ on Γ_{13}, and hence,

$$\kappa(x_0) \equiv \int_{\Gamma_{13}} \frac{\partial U}{\partial n}\, ds > 0.$$

Note, finally, that in the special case $k_1(\cdot) = k_2(\cdot)$ of a single component, the exchange term takes the form

$$\Gamma(p_1, p_2) = \kappa(x_0) \int_{p_1}^{p_2} \rho k\, ds = \kappa(x_0)\tilde{\rho k}(p_2 - p_1)$$

in which $\tilde{\rho k}$ is the *average fluid conductivity* over the indicated pressure interval.

2.2. **The Boundary Conditions.** The boundary of Ω is given by the disjoint union of the parts Γ_D and Γ_{fl}. The boundary of Ω is also written as another disjoint union of the parts Γ_0 and Γ_{tr}. Finally, we define a pair of functions, $\beta_j(\cdot)$, $j = 1, 2$, on that portion of the boundary, $\Gamma_S = \Gamma_{fl} \cap \Gamma_{tr}$, which is neither drained nor clamped, and these functions specify the surface fraction of the pores in the j-th component which are *sealed* along Γ_S. For these pores, the pressure $p_j(\cdot)$ continues to contribute to the total stress within the matrix. The remaining portion $1 - \beta_j(\cdot)$ of the pores are *exposed* along Γ_S, and these contribute to the flux but not to the stress. Thus, on any portion of Γ_S which is completely exposed, that is, where $\beta_j = 0$, the fluid pressure does not contribute to the support of the matrix, so only the *effective* or elastic component of stress is specified for that component. Similarly, on any portion which is completely sealed, that is, where $\beta_j = 1$, the total stress contains the full available pressure $\alpha_j \chi_j(p_j) p_j$, and there is no production of boundary flux from that component resulting from dilation. The classical models have considered *only* this latter case, even in the simplest homogeneous and linear models.

We can now state the boundary conditions for our problem. On the Dirichlet boundary Γ_D, the value of the two pressures is given by the depth below the surface:

$$p_j(x,t) = d(x), \quad x \in \Gamma_D, \quad j = 1, 2, \tag{2a}$$

where $d(\cdot) > 0$. Along Γ_{fl} the *boundary flux* is given by

$$q_j \equiv -\frac{\partial}{\partial t}[(1 - \beta_j)\alpha_j \mathbf{u} \cdot \mathbf{n}] - \rho(p_j) k_j(p_j) \nabla p_j \cdot \mathbf{n}\,,$$

for $j = 1, 2$, where $\mathbf{n}$ is the unit outward normal on the boundary, $\partial\Omega$. On the Neumann boundary Γ_N, there is no flow, so we have a null normal flux from each component:

$$q_j(x,t) = 0, \quad x \in \Gamma_N, \quad j = 1, 2. \tag{2b}$$

On the *unilateral boundary* Γ_U, we have

$$p_j \le 0,\ q_j \ge 0,\ p_j\, q_j = 0, \quad x \in \Gamma_U, \quad j = 1, 2. \tag{2c}$$

The remaining boundary conditions on $\partial\Omega$ involve the displacements and *tractions*, namely,

$$\mathbf{u} = \mathbf{0} \text{ on } \Gamma_0\,, \quad \sigma(x,t)\mathbf{n} = \mathbf{f} \text{ on } \Gamma_{tr}\,, \tag{2d}$$

where the traction forces are given by

$$\sigma_{ij}(x,t)n_j \equiv \sum_{j=1}^{3} \sigma'_{ij} n_j - (\beta_1 \alpha_1 \chi_1(p_1) p_1 + \beta_2 \alpha_2 \chi_2(p_2) p_2) n_i \,.$$

2.3. **The Initial Conditions.** Finally, we shall require that the initial value of the fluid content $\theta_j(\cdot)$ be specified in each component,

$$\varphi_j(p_j(x,0)) S_j(p_j(x,0)) \rho(p_j(x,0)) + \alpha_j \nabla \cdot \mathbf{u}(x,0) = \theta_j(x)\,, \quad x \in \Omega\,,\ j = 1,2, \tag{3}$$

where the initial displacement and pressure satisfy an initialization constraint.

2.4. **The Semi-Linear Case.** Assume that there are constants $\lambda_j > 0$ for which

$$\lambda_j (p\,\chi_j(p))' = \rho(p)\,k_j(p)\,, \quad p \in \mathbb{R}\,. \tag{4}$$

These relate the Bishop parameters $\chi_j(\cdot)$ to the fluid conductivity $\rho(\cdot)\,k_j(\cdot)$. Since this product is positive, it follows that $p\,\chi_j(p)$ is monotone. Furthermore, when $\rho(\cdot)\,k_j(\cdot)$ is monotone, it follows that $p\,\chi_j(p)$ is convex, so $\chi_j(\cdot)$ is monotone. Note that our assumption (4) requires that the pressure stress is given by

$$\lambda_j \nabla (p\,\chi_j(p)) = \rho(p)\,k_j(p)\,\nabla p\,,$$

i.e., the pressure component of the Darcy velocity. This relates the flux to the *viscous resistance* of the medium to the fluid flow. Moreover, it permits the Bishop parameter to have essentially the same form as the saturation function, and this seems to be all the information we have about the Bishop parameter.

This assumption (4) permits a change of variable in which the system is essentially a semilinear system. For this system we have proved *existence* of a weak solution, and it includes gravity in addition to the six remaining additional sources of nonlinearity. The proof follows the same technique as that of [26], namely, we phrase the problem as an abstract Cauchy problem for an implicit evolution equation for which we use use monotonicity and compactness methods [14, 23]. We are extending this result to include the degenerate case of [30] in which there is no diffusion between isolated blocks, so $K_2(\cdot) = 0$, as well as the pseudo-parabolic *fissured medium equation* in which also there is negligible storage in the fissures, hence, $\varphi_1(\cdot) = 0$.

References

[1] H.W. Alt and S. Luckhaus. Quasilinear elliptic-parabolic differential equations. *Math. Z.*, 183:311–341, 1983.

[2] H.W. Alt, S. Luckhaus, and A. Visintin. On nonstationary flow through porous media. *Ann. Mat. Pura Appl.*, 136(4):303–316, 1984.

[3] J.-L. Auriault and E. Sanchez-Palencia. 'Étude du comportement macroscopique d'un milieu poreux saturé déformable'. *Journal de Mécanique*, 16:575–603, 1977.

[4] M. Bai, D. Elsworth, and J. Roegiers. Multiporosity/multipermeability approach to the simulation of naturally fractured reservoirs. *Water Resources Research*, 29:1621–1633, 1993.

[5] C. Baiocchi. Su un problema di frontiera libera connesso a questioni di idraulica. *Ann. Mat. Pura Appl.*, 92(4):107–127, 1972.

[6] G. I. Barenblatt, I. P. Zheltov, and I. N. Kochina. Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. *J. Appl. Mech.*, 24:1286–1303, 1960.

[7] J. Bear. *Dynamics of Fluids in Porous media.* American Elsevier, New York, 1972.

[8] J. Bear. Modeling flow and contaminant transport in fractured rocks. In J. Bear, C.-F. Tsang, and G. de Marsily, editors, *Flow and Contaminant Transport in Fractured Rock.* Academic Press, New York, 1993.

[9] J. Berryman and H. F. Wang. The elastic coefficients of double-porosity models for fluid transport in jointed rock. *J. Geophys. Res.*, 100:24611–24627, 1995.

[10] D. E. Beskos and E. C. Aifantis. On the theory of consolidation with double porosity. *Int. J. Engng. Sci.*, 20:1079–1094, 1982.

[11] D. E. Beskos and E. C. Aifantis. On the theory of consolidation with double porosity, ii. *Int. J. Engng. Sci.*, 24:1697–1716, 1986.

[12] M. Biot. General theory of three-dimensional consolidation. *J. Appl. Phys.*, 12:155–164, 1941.

[13] M. Biot. Theory of elasticity and consolidation for a porous anisotropic solid. *J. Appl. Phys.*, 26:182–185, 1955.

[14] E. DiBenedetto and R. E. Showalter. Implicit degenerate evolution equations and applications. *SIAM J. Math. Anal.*, 12:731–751, 1981.

[15] G. Gilardi. A new approach to evolution free boundary problems. *Comm. Partial Differential Equations*, 4:1099–1122, 1979.

[16] W.G. Gray and B.A. Schrefler. Thermodynamic approach to effective stress in partially saturated porous media. *European Journal of Mechanics, A/Solids*, 20:521–538, 2001.

[17] Ulrich Hornung. A parabolic-elliptic variational inequality. *Manuscripta Mathematica*, 39:155–172, 1982.

[18] P. S. Huyakorn and G. F. Pinder. *Computational Methods in Subsurface Flow.* Academic Press, New York, 1983.

[19] K. T. Lewallen and H. F. Wang. Consolidation of a double-porosity medium. to appear.

[20] A. Mourits and F. M. Settari. Coupling of geomechanics and reservoir simulation models. In Siriwardane and Zaman, editors, *Computer Methods and Advances in Geomechanics*, pages 2151–2158. Balkema, Rotterdam, 1994.

[21] J. R. Rice and M. P. Cleary. Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents. *Rev. Geophysics and Space Phy.*, 14:227–241, 1976.

[22] A.P.S. Selvadurai, editor. *Mechanics of Poroelastic Media.* Kluwer Acad. Pub., Dordrecht, 1996.

[23] R. E. Showalter. *Monotone Operators in Banach Space and Nonlinear Partial Differential Equations.* Number 49 in Mathematical Surveys and Monographs. American Mathematical Society, Providence, 1997.

[24] R. E. Showalter. Diffusion in poro-elastic media. *Jour. Math. Anal. Appl.*, 251:310–340, 2000.

[25] R. E. Showalter and B. Momken. Single-phase flow in composite poro-elastic media. *Math. Methods in the Applied Sciences*, 25:115–139, 2002. TICAM Report 00-31.

[26] R. E. Showalter and N. Su. Partially saturated flow in a poroelastic medium. *Discrete Cont. Dynamic Systems - Series B*, 1:403–420, 2001. TICAM Report 01-01.

[27] A. Torelli. Sur un problema a frontiera libera di evoluzione. *Boll. Un. Mat. Ital.*, 11:559–570, 1975.

[28] S. Valliappan and N. Khalili-Naghadeh. Flow through fissured porous media with deformable matrix. *Int. J. Numerical Methods in Engineering*, 29:1079–1094, 1990.

[29] A. Visintin. Existence results for some free boundary filtration problems. *Ann. Mat. Pura Appl.*, 124(4):293–320, 1980.

[30] J. E. Warren and P. J. Root. The behavior of naturally fractured reservoirs. *Soc. Petr. Eng. J.*, 3:245–255, 1963.

[31] R. K. Wilson and E. C. Aifantis. On the theory of consolidation with double porosity. *Int. J. Engineering Sciences*, 20:1009–1035, 1982.

[32] A. Zenisek. Finite element methods for coupled thermoelasticity and coupled consolidation of clay. *RAIRO Anal. Numér.*, 18:183–205, 1984.

[33] O.C. Zienkiewicz, A.H.C. Chan, M. Pastor, B.A. Schrefler, and T. Shiomi. *Computational Geomechanics*. Wiley, Chichester, 1999.

[34] O.C. Zienkiewicz, C.T. Chang, and P. Battess. Drained, undrained, consolidating, and dynamic behaviour assumptions in soils, limits of validity. *Geotechnique*, 30:385–395, 1980.

Department of Mathematics, University of Texas at Austin, Austin, TX 78712, USA
E-mail address: show@math.utexas.edu

Department of Mathematical Sciences, Tsinghua University, Beijing 100084, PRC
E-mail address: nsu@math.tsinghua.edu.cn