PORO-PLASTIC FILTRATION COUPLED TO STOKES FLOW

R. E. SHOWALTER

ABSTRACT. We report on the model development and mathematical analysis of the exchange of fluid and stress between a Biot model of an elastic-plastic porous structure saturated with a slightly compressible viscous fluid coupled to the Stokes flow in an adjacent open channel. The coupled systems of partial differential equations and interface conditions will be formulated in a mixed variational setting and resolved by nonlinear semigroup methods.

1. INTRODUCTION

Visco-elastic and elastic-plastic behaviors of porous media are common, especially in soils, and generally for materials of interest in geomechanics. Various approaches to include appropriate constitutive equations have been developed, and many numerical codes include plasticity models [14], [15], [27], [37], [48]. Another example arises from the irreversible fluid content that corresponds to the *plastic porosity* of the porous medium, and these examples illustrate the need for the inclusion of *hysteresis* in models of deforming porous media. Such memory dependent phenomena as secondary viscosity and hysteresis effects in models of flow and transport in (rigid) porous media have been studied by *nonlinear semigroup techniques* as in [40], [47]. In recent work of [41], we have developed a very general model for Darcy flow through a *viscous-plastic* medium in the form of the coupled system of partial differential and functional equations

$$\frac{\partial}{\partial t}\big(c_1\, p^1 + c_0 \boldsymbol{\nabla}\cdot \mathbf{u}^1\big) - \boldsymbol{\nabla}\cdot k(\boldsymbol{\nabla} p^1) = h_1, \tag{1.1a}$$

$$\rho_1 \frac{\partial^2}{\partial t^2}\mathbf{u}^1 - \boldsymbol{\nabla}\cdot\sigma^1 + c_0 \nabla p^1 = \mathbf{f}_1, \tag{1.1b}$$

$$\sigma^1 = \mathbf{H}(\varepsilon(\mathbf{u}^1))\,. \tag{1.1c}$$

The system (1.1) is of *Biot* type [9], [10], [11] and consists of the diffusion equation for the pressure, the conservation equation for momentum, and a constitutive relation for the deformation response of the medium, respectively. The constitutive relation (1.1c) involves the stress σ^1 and the *small strain* tensor $\varepsilon(\mathbf{u}^1)$. The symmetric derivative of a vector function $\mathbf{v}(x)$ is the tensor $\varepsilon_{ij}(\mathbf{v}) \equiv \frac{1}{2}(\partial_i v_j + \partial_j v_i)$.

The slow flow through an adjacent open channel, possibly a macropore, an isolated cavity, or a connected fracture system, is described by the *compressible Stokes system* [44], [36]

$$\frac{\partial}{\partial t}(c_2 p^2) + \boldsymbol{\nabla}\cdot\mathbf{v}^2 + c_2\rho_2\mathbf{g}\cdot\mathbf{v}^2 = 0\,, \tag{1.2a}$$

$$\frac{\partial}{\partial t}(\rho_2\mathbf{v}^2) - \boldsymbol{\nabla}\cdot\sigma^2 + \boldsymbol{\nabla} p^2 = c_2\rho_2\mathbf{g}\, p^2\,, \tag{1.2b}$$

$$\sigma^2_{ij} = \lambda_2\delta_{ij}\varepsilon(\mathbf{v}^2)_{kk} + 2\mu_2\varepsilon(\mathbf{v}^2)_{ij}\,. \tag{1.2c}$$

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In the limiting case of an *incompressible* fluid, we have $c_2 = 0$ and classical *Stokes flow*,

$$\boldsymbol{\nabla} \cdot \mathbf{v}^2 = 0\,, \qquad \tfrac{\partial}{\partial t}(\rho_2 \mathbf{v}^2) - \mu_2 \Delta \mathbf{v}^2 + \boldsymbol{\nabla} p^2 = \mathbf{0}\,.$$

The Biot system (1.1) includes a very extensive variety of models. Moreover, the theory developed in [41] permits highly degenerate situations in which some (or even all) of the parameters c_1, ρ_1, and k may vanish! Specifically, we include any combination of the *quasi-static* case, $\rho_1 = 0$, the *incompressible* case, $c_1 = 0$, the *uncoupled* case, $c_0 = 0$, and even the *impermeable* case, $k = 0$. The porous solid is a very general rheological material made up of the parallel combination of elementary components of various types, elastic, viscous, and plastic, with combinations of kinematic and isotropic hardening. Such a construction requires the introduction of *internal variables* [19], [42], [47].

Our objective is to formulate a model of a composite poro-mechanical system which accurately characterizes the fluid exchange and stress balance between the elastic-plastic porous medium and a contiguous fluid-filled chamber, and to show that this model leads to a mathematically well-posed problem which is amenable to analysis and computation. The interface coupling conditions include the continuity of the normal fluid flux and of stress. Two additional constitutive relations concern the dependence of the Darcy flux at the interface on the pressure increment and the effect of the tangential component of stress on the velocity increment at the interface. The former is a classical *Robin*-type condition, and the latter is the slip condition of *Beavers-Joseph-Saffman.*

2. THE BIOT-STOKES SYSTEM

Suppose that disjoint regions Ω_1 and Ω_2 in $\mathbb{R}^3$ share the common *interface*, $\Gamma_{12} = \partial\Omega_1 \cap \partial\Omega_2$. The first region Ω_1 is the *porous matrix* structure, and the second region Ω_2 is the adjacent *macro-void system.* Denote by $\mathbf{n}$ the unit normal vector on the boundaries, directed *out* of Ω_1 and *into* Ω_2. The derivative with respect to time will be denoted by a superscript dot, so $\mathbf{v}^1(x,t) = \dot{\mathbf{u}}^1(x,t)$ denotes the *velocity* corresponding to a *displacement* $\mathbf{u}^1(x,t)$ of the *porous structure* at $x \in \Omega_1$. Let $\mathbf{v}^2(x,t)$ be the velocity of the *fluid* at $x \in \Omega_2$. The *fluid pressure* in Ω_1 is $p^1(x,t)$ and in the adjacent channel system Ω_2 is $p^2(x,t)$.

The mechanical behavior of the porous solid is determined by classical small-strain elasticity. Boldface letters indicate vectors in $\mathbb{R}^3$ and Greek letters are used for symmetric second-order tensors. Repeated indices are summed, so the scalar product of two vectors is $\mathbf{v} \cdot \mathbf{w} = v_i w_i$, and that of two second-order tensors is $\sigma : \tau = \sigma_{ij}\tau_{ij}$. Let $\mathbf{n} = \{n_i\}$ be the unit normal vector on a surface. For a vector $\mathbf{w}$, we denote the normal projection $w_n = \mathbf{w} \cdot \mathbf{n}$ and the tangential component $\mathbf{w}_T = \mathbf{w} - w_n \mathbf{n}$. Similar notation is used for tensors.

2.1. *The System*

We shall write the constitutive equations together with the conservation equations for mass and momentum balance as a system of first-order partial differential equations in each of the two regions. The constitutive laws are written in the differential form $\mathbb{M}(\dot{\sigma}^1) + \mathbb{L}(\sigma^1) \ni \varepsilon(\mathbf{v}^1)$ in Ω_1 for the *stress* σ^1 corresponding to the *strain rate* $\varepsilon(\mathbf{v}^1)$ in the porous structure and $\sigma^2_{ij} = \lambda_2 \delta_{ij} \varepsilon(\mathbf{v}^2)_{kk} + 2\mu_2 \varepsilon(\mathbf{v}^2)_{ij}$ in Ω_2 for the *viscous stress* corresponding to the *strain rate* $\varepsilon(\mathbf{v}^2)$ of the *Newtonian fluid.* For a purely elastic structure, $\mathbb{L} = 0$

and $\mathbb{M}$ is the *compliance* tensor. For plasticity models, $\mathbb{L}(\cdot)$ is a *variational inequality.* Note that $\sigma^1 - c_0\, p^1\, \delta$ is the *total stress* due to deformation and pore pressure p^1 within the matrix, and $\sigma^2 - p^2\,\delta$ is the combined viscous and pressure stress of the fluid. Here both p^1 and p^2 are the thermodynamic pressure of the barotropic fluid in the respective regions. The *Biot-Stokes system* takes the form

$$c_1\dot{p}^1 + \boldsymbol{\nabla}\cdot\mathbf{q} + c_0\boldsymbol{\nabla}\cdot\mathbf{v}^1 = h_1\,, \tag{2.1a}$$

$$\mathcal{Q}(\mathbf{q}) + \boldsymbol{\nabla}p^1 = 0\,, \tag{2.1b}$$

$$\rho_1\dot{\mathbf{v}}^1 - \nabla\mu^*(\boldsymbol{\nabla}\cdot\mathbf{v}^1) - \boldsymbol{\nabla}\cdot\sigma^1 + c_0\boldsymbol{\nabla}p^1 = \mathbf{f}_1\,, \tag{2.1c}$$

$$\mathbb{M}(\dot{\sigma}^1) + \mathbb{L}(\sigma^1) \ni \varepsilon(\mathbf{v}^1) \text{ in } \Omega_1\,, \text{ and} \tag{2.1d}$$

$$c_2\dot{p}^2 + \boldsymbol{\nabla}\cdot\mathbf{v}^2 + c_2\rho_2\mathbf{g}\cdot\mathbf{v}^2 = h_2\,, \tag{2.1e}$$

$$\rho_2\dot{\mathbf{v}}^2 - \boldsymbol{\nabla}\cdot\sigma^2 + \boldsymbol{\nabla}p^2 - c_2\rho_2\,p^2\,\mathbf{g} = \mathbf{f}_2\,, \tag{2.1f}$$

$$\mathcal{C}^2\sigma^2 - \varepsilon(\mathbf{v}^2) = 0 \text{ in } \Omega_2\,. \tag{2.1g}$$

The first (2.1a) is the *storeage equation* for the fluid mass conservation in the pores of the matrix in which the *flux* $\mathbf{q}$ is the relative velocity of the fluid within the porous structure given by *Darcy's law.* This is written in the form (2.1b) of a force balance in which the flow resistance tensor $\mathcal{Q}$ is the inverse of the conductivity tensor k_{ij}. The third set of equations (2.1c) is the standard *Navier* system for the conservation of momentum of the matrix structure, and $\mu^* \geq 0$ arises from *secondary consolidation* effects. The constitutive relation (2.1d) is the *differential form* of the elastic-plastic stress-strain relationship. These first four equations are equivalent to the Biot system (1.1), and they can be generalized to include *quasilinear diffusion* $\mathcal{Q}(\cdot)$ and *nonlinear dissipation* $\mu^*(\cdot)$ in addition to the highly nonlinear constitutive relation (2.1d).

The last three equations are just the compressible Stokes system (1.2) for pressure $p^2(x,t)$ and velocity $\mathbf{v}^2(x,t)$ of the fluid. The equation (2.1e) accounts for the fluid mass conservation in the channel, and (2.1f) is the momentum conservation equation. The gravitational force $\mathbf{g}$ contributes to both of these. The *Newtonian fluid* is described by the constitutive relation (2.1g) in which the tensor $\mathcal{C}^2$ is the inverse to the viscosity tensor.

2.2. *Boundary and Interface Conditions*

We choose the *boundary conditions* on $\partial\Omega_1\cup\partial\Omega_2-\Gamma_{12}$ in a classical simple form, since they play no essential role here. On the exterior boundary of the porous medium, $\partial\Omega_1 - \Gamma_{12}$, we shall impose *drained conditions* $p_1 = 0$ on fluid pressure and the *clamped condition* $\mathbf{v}_1 = \mathbf{0}$ on velocity of the structure. On the exterior boundary of the free fluid, $\partial\Omega_2 - \Gamma_{12}$, we shall impose the *no-slip condition* $\mathbf{v}_2 = \mathbf{0}$ on fluid velocity. More general conditions can be given as in [41].

In order to complete a well-posed problem, additional *interface conditions* must be imposed across the interior boundary Γ_{12}. Let's begin by reviewing the interface conditions that have been used previously to couple various models of fluid and solid composites.

2.2.1. *Fluid–Solid Contact*

The natural transmission conditions at the interface of a free fluid and an impervious solid consist of the continuity of displacement and of stress [35]. The effective flow through a rigid micro-porous and permeable matrix is described by the *Darcy law*, $q_i = -k_{ij}\partial_j p^1$, where $\mathbf{q}$ is the filtration velocity or flux of fluid driven by a pressure gradient, and k_{ij} is the *conductivity*. In fact, Darcy's law can be realized as the upscaled limit by averaging or *homogenization* of a fine-scale periodic array of a rigid solid and intertwined fluid. See [43], [1], [20]. Similar results are obtained when the solid is permitted to be *elastic*, and then various scalings of the viscosity lead to a *viscous solid* or to the *Biot model of poroelasticity* (1.1). See [6], [34], [36], [13], [46], [7], [17], [5], [45].

2.2.2. *Fluid–Porous Medium*

The description of a free fluid in contact with a rigid but porous matrix requires a means to couple the fluid flow to the upscaled Darcy filtration. Since a Stokes system is used for the free fluid, we have two distinct scales of hydrodynamics, and these are represented by two completely different systems of partial differential equations. Fluid conservation is a natural requirement at the interface, and other classically assumed conditions such as continuity of pressure or vanishing tangential velocity of the viscous fluid have been investigated [16], [25], but these issues have been controversial. See the discussion on p. 157 of [36]. In fact, one can even question the *location* of the interface, since the porous medium itself is already a mixture of fluid and solid. Moreover, it was reported in [8] that fluid in contact with a porous medium flows faster along the interface than a fluid in contact with a solid surface: there is a substantial *slip* of the fluid at the interface with a porous medium. It was proposed that the normal derivative of the tangential component of fluid velocity $\mathbf{v}_T$ satisfy

$$\tfrac{\partial}{\partial n}\mathbf{v}_T = \tfrac{\gamma}{\sqrt{K}}(\mathbf{v}_T - \mathbf{q}_T)$$

where K is the permeability of the porous medium, and γ is the *slip rate coefficient.* This condition was developed further by [32] and [23], and a substantial rigorous analysis of such interface conditions was given by [21], [22]. See [30], [26] for excellent discussions, [33], [18], [24], [2], [3] for numerical work, [31] for dependence on the slip parameter, and [4] for homogenization results on related problems.

2.2.3. *Fluid–Elastic Porous Medium*

Any model of free fluid in contact with a *deformable* and porous medium contains the upscaled filtration velocity in addition to the displacement and stress variations of the porous matrix. These must be coupled to the Stokes flow, so all of the previous issues are present in the interface conditions. See [28], [29], [39].

2.2.4. *Fluid–Elastic-Plastic Porous Medium*

We begin with the mass-conservation requirement that the normal fluid flux be continuous across the interface. For this purpose, we introduce the parameter β which represents the surface fraction of the interface on which the diffusion paths of the structure are *sealed.* The remaining fraction $1-\beta$ is the *contact surface* along Γ_{12}, where the diffusion paths of the porous medium are exposed to the fluid in the open channel, and so the motion of the structure contributes to the interfacial fluid mass flux. Thus, the solution is required to satisfy the *admissability constraint*

$$(c_0(1-\beta)\mathbf{v}^1 + \mathbf{q}) \cdot \mathbf{n} = \mathbf{v}^2 \cdot \mathbf{n} \tag{2.2a}$$

for the conservation of fluid mass across the interface. We shall assume that the Darcy flow across Γ_{12} is driven by the difference between the total normal stress of the fluid and the pressure internal to the porous medium according to

$$\sigma_n^2 - p^2 + p^1 = \alpha \mathbf{q} \cdot \mathbf{n}\,. \tag{2.2b}$$

The constant $\alpha \geq 0$ is the *fluid entry resistance.* The conservation of momentum requires that the total stress of the porous medium is balanced by the total stress of the fluid. For the normal component this means

$$\sigma_n^1 - c_0 p^1 = c_0(1-\beta)(\sigma_n^2 - p^2)\,, \tag{2.2c}$$

and for the tangential component we have

$$\sigma_T^1 = \sigma_T^2\,. \tag{2.2d}$$

Finally, this common tangential stress is assumed to be proportional to the *slip rate* according to the Beavers-Joseph-Saffman condition

$$\sigma_T^2 = \gamma\sqrt{\mathcal{Q}}(\mathbf{v}_T^2 - \mathbf{v}_T^1)\,. \tag{2.2e}$$

We have shown that the *interface conditions* (2.2) suffice precisely to couple the Biot system (1.1) in Ω_1 to the Stokes system (1.2) in Ω_2.

3. THE INITIAL-VALUE PROBLEM

The system (2.1) with (2.2) can be written in the form

$$\mathbf{v}(t) \in \mathbb{V}: \ \frac{d}{dt}(\mathcal{A}\mathbf{v}(t)) + \mathcal{B}\mathbf{v}(t) = \mathbf{f}(t) \text{ in } \mathbb{H}',\ t > 0, \tag{3.1a}$$

in appropriate function spaces $\mathbb{V}$ and $\mathbb{H}$ where the linear operator $\mathcal{A}: \mathbb{H} \to \mathbb{H}'$ is degenerate, symmetric and nonnegative, and the nonlinear $\mathcal{B}: \mathbb{V} \to \mathbb{V}'$ is monotone. These operators are defined below. The evolution equation (3.1a) is to be solved subject to the *initial condition*

$$\mathcal{A}\mathbf{v}(0) = \mathcal{A}\mathbf{v}_0\,. \tag{3.1b}$$

The equation (3.1a) is an example of an *implicit evolution equation* with *degenerate* operators as coefficients, sometimes known as a degenerate *Sobolev equation*; see [38].

3.1. *The mixed formulation*

Here we consider the system (2.1), (2.2), but re-order the variables according to their role in the *physics* of the model, not in the *geometry* of the problem. Thus, we separate the variables into two spaces. The first space $\mathbf{X}$ consists of *admissable velocities*, $\mathbf{X} = \{[\mathbf{q}, \mathbf{v}_1, \mathbf{v}_2] :\ (c_0(1-\beta)\mathbf{v}^1 + \mathbf{q}) \cdot \mathbf{n} = \mathbf{v}^2 \cdot \mathbf{n}\}$, and the second space $\mathbf{Y}$ contains the *generalized stresses*, $\mathbf{Y} = \{[p_1, \sigma_1, p_2, \sigma_2]\}$. We define the operators

$$A:\ \mathbf{X} \to \mathbf{X}' \quad B:\ \mathbf{X} \to \mathbf{Y}' \quad C:\ \mathbf{Y} \to \mathbf{Y}'$$

by means of the matrix operators

$$A = \begin{pmatrix} \mathcal{Q}(\cdot) + \gamma_n' \alpha \gamma_n & 0 & 0 \\ 0 & \gamma_T' \gamma \sqrt{\mathcal{Q}} \gamma_T & -\gamma_T' \gamma \sqrt{\mathcal{Q}} \gamma_T \\ 0 & -\gamma_T' \gamma \sqrt{\mathcal{Q}} \gamma_T & \gamma_T' \gamma \sqrt{\mathcal{Q}} \gamma_T \end{pmatrix},$$

$$B = \begin{pmatrix} \delta : \varepsilon & c_0 \delta : \varepsilon & 0 \\ 0 & -\varepsilon & 0 \\ 0 & 0 & \delta : \varepsilon + c_2 \rho_2 \mathbf{g} \cdot \\ 0 & 0 & -\varepsilon \end{pmatrix},$$

$$C = \mathrm{diag}(0,\ \mathbb{L}(\cdot),\ 0,\ \mathcal{C}^2)$$

$$D_1 = \mathrm{diag}(0,\ \rho_1,\ \rho_2), \qquad D_2 = \mathrm{diag}(c_1,\ \mathbb{M},\ c_2,\ 0)\,,$$

where γ_n and γ_T denote normal and tangential trace on the interface. Then the system (2.1), (2.2) takes the form

$$D_1 \dot{\mathbf{x}} + A\mathbf{x} - B'\mathbf{y} = \mathbf{f}\,,$$
$$D_2 \dot{\mathbf{y}} + B\mathbf{x} + C\mathbf{y} = \mathbf{g}\,,$$

for the unknowns $\mathbf{x} \equiv [\mathbf{q}, \mathbf{v}_1, \mathbf{v}_2] \in \mathbf{X},\ \mathbf{y} \equiv [p_1, \sigma_1, p_2, \sigma_2] \in \mathbf{Y}$. Now set

$$\mathcal{A} = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix} \qquad \mathcal{B} = \begin{pmatrix} A & -B' \\ B & C \end{pmatrix}$$

to get the system into the form (3.1a) on $\mathbb{V} = \mathbf{X} \times \mathbf{Y}$.

3.2. *Remarks*

The means by which we establish the solvability of the system will depend critically on how much degeneracy occurs in the operators. For example, in the *least degenerate* case in which all the constants c_1, ρ_1, c_2, ρ_2 are strictly positive, the resolution is straightformward. In the mathematically more interesting and practically more relevant situations, some of these coefficients will vanish. In many of these cases, we can eliminate appropriate variables, thereby obtaining elliptic terms in the system, and then solve the reduced higher order system.

This mixed formulation requires a *closed range condition* on the operator B, and it provides a natural and well established approach to the *numerical approximation* of the problem; see [12]. In addition, the analysis of this formulation provides a means to establish the relation with the *singular limits* such as the incompressible case $c_2 = 0$ of the Stokes flow and the *quasistatic* case $\rho_1 = 0$ of consolidation processes. These issues will be developed for nonlinear extensions of these models in forthcoming works.

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Department of Mathematics, Oregon State University, Corvallis, OR 97331-4605
E-mail address: show@math.oregonstate.edu