

PREFACE

Analysis and Approximation of Microstructure Models

Experimental observations indicate that the multiscale structure of materials has a strong effect on their effective (macroscopic) properties. The challenge is to develop mathematical models that incorporate the fine-scale effects of the microstructure and accurately represent the physical phenomena at spatial scales amenable to computation and analysis. For real-world applications and their numerical simulation, the choice of an appropriate upscaled model is essential. The estimates of error between solutions of the exact microscopic models and corresponding averaged models provide crucial information on the approximation rate to the original processes.

This special issue includes eight papers dealing with the multiscale modeling and analysis of reaction, transport (diffusion, drift) and mechanics in media with either periodic or non-periodic microstructures. Special attention is devoted to the understanding of boundary layers caused by the higher order approximation of the macroscopic solution, the derivation of bounds for effective coefficients and correctors/error estimates on significant averaged quantities, as well as to the asymptotic analysis of spectral problems and effective numerical computation for generalized polarization tensors.

- G. A. Chechkin and T. A. Mel'nyk study the *Asymptotics of eigenvalues to spectral problem in thick cascade junction with concentrated masses*. The influence of domain perturbation given by junctions of finite and of small length and density perturbation on the asymptotic behaviour of the eigenvalues and eigenfunctions is depicted in five qualitatively different cases. The method of matching of asymptotic expansions is used to derive the homogenized spectral problem and correctors for the eigenfunctions.
- *Burnett coefficients and laminates* are the subject of C. Conca, J. San Martín, L. Smaranda M. Vanninathan. For a given elliptic operator with periodic coefficients in a divergence form, Burnett coefficients (the forth order dispersion tensor) can be defined as forth order terms in the Taylor expansion of the principal Bloch eigenvalue about zero. The lower and upper bounds for this tensor are obtained. The spectral decomposition of the dispersion tensor in laminates is studied using blossoming principle.
- The periodic unfolding method is applied by B. Cabarrubias and P. Donato for the *Homogenization of a quasilinear elliptic problem with nonlinear Robin boundary conditions*. The pointwise estimate obtained for the difference of two homogenized matrices for perforated domains in terms of the difference of the original ones ensures the uniqueness of the macroscopic problem.

- The focus of D. Onofrei and B. Vernescu is on the *Asymptotic analysis of second-order boundary layer correctors*. The ideas related to the periodic unfolding method are used to improve the H^1 -error estimates assuming only the boundedness for coefficients of the considered elliptic problem.
- Starting from a scenario describing the transport of sulfate ions attacking cement-based materials, T. Fatima, A. Muntean and M. Ptashnyk derive *Unfolding-based corrector estimates for a reaction-diffusion system predicting concrete corrosion*. The main advantage of using the unfolding technique in the proof of corrector estimates is that only H^1 -regularity is required of solutions of microscopic equations and of unit cell problems.
- I. C. Kim gives an overview around a particular technique on *Homogenization and error estimates of free boundary velocities in periodic media*. The construction of appropriate viscosity sub- and super-solutions is essential in the definition of the limiting free boundary velocity and for the derivation of error estimates.
- *Numerical computation of approximate generalized polarization tensors* is developed by Y. Capdeboscq, A. B. Karrman and J.C. Nédélec. These tensors are the coefficients that arise in the multipolar expansion of the steady-state voltage perturbation caused by an inhomogeneity in the conductivity. Special attention is devoted to the construction of a simple algorithm to compute approximate values of the contracted generalized polarization tensor.
- Inspired by existing approaches to the homogenization of Schrödinger-like equations with large drift, H. Douanla and N. Svanstedt investigate in *Homogenization of a nonlinear elliptic problem with large nonlinear potential* the case of the p -Laplace equation in the presence of a suitably scaled potential field. The two-scale convergence and the pseudo-monotonicity of the nonlinear operator are applied to derive macroscopic equations.

The multiscale nature of biological and physical systems gives rise to many interesting and challenging mathematical problems. In this issue, the reader will find not only a wide spectrum of multiscale analysis results (like convergence proofs), but also such practically important information as bounds on effective coefficients, error estimates between the microscopic and upscaled solutions to partial differential equations, or a code (with detailed implementation hints) for the computation of generalized polarization tensors for the reconstruction of conductivities.

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