

HW BIFURCATIONS

~~1. For ex~~
~~1. (a)~~

1. For the following ODE, determine the equilibrium solutions & examine their stability

$$\frac{du}{dt} = u(9 - \mu u)(\mu + 2u - u^2)$$

& sketch the branching diagram.

2. Discuss the nature of the singular points for $f(\mu, u) = (u - \mu^2)^2 - \mu^5 = 0$

3. Bead on rotating hoop. See class notes:

$$mR\ddot{\theta} = mR\omega^2 \cos\theta \sin\theta - mg \sin\theta$$

(a) Find equilibrium points $\begin{cases} \theta_0 = 0 \\ \theta_1 = \arccos(g/R\omega^2) \end{cases}$

(b) let $\theta = \theta_0 + \delta\theta$, where $\theta_0 = 0$. Assume $|\delta\theta| \ll 1$
and

$$\sin x \approx x, \cos x \approx 1 \text{ for small } x.$$

Solve the dynamics for $\delta\theta$ and show that

$$\delta\theta = A \cos \sqrt{\frac{g}{R} - \omega^2} t + B \sin \sqrt{\frac{g}{R} - \omega^2} t$$

with $\omega < \sqrt{g/R}$



(c) Discuss stability of θ_0

(d) Assume $\theta = \theta_1 + \delta\theta$ and describe stability of bead on rotating loop for $\omega > \sqrt{g/R}$.

4. Prove \$ Lemma