HW Regular Perturbations

① Verify that $\int_0^{\varepsilon} e^{-x^2}\,dx = O(\varepsilon)$ as $\varepsilon \to 0^+$

② Verify that $e^{-\varepsilon} = o(1)$ as $\varepsilon \to \infty$

③ Use Poincaré-Lindstedt Method to find a 2-term approximation to the solution of

$$\ddot{u} + u = \varepsilon(1-u^2)\dot{u}$$

④ Use a regular perturbation series to find a 2-term approximation to the solution of

$$\ddot{u} + u = \varepsilon(1-u^2)\dot{u}$$

⑤ For $\begin{cases} \dfrac{dy}{dt} = 1 + (1+\varepsilon)\,y^2 & t > 0 \quad \varepsilon \ll 1 \\ y(0) = 1 \end{cases}$

(a) Solve exactly

(b) Find 2-term approx via regular series expansion

(c) Find first two terms of series expansion of exact solution found in (a) and compare to (b) approximation

(6) Find the roots of

$$(*) \quad x^3 - 4.001x + 0.002 = 0$$

One method is to write (*) as

$$x^3 - (4+\varepsilon)x + 2\varepsilon = 0$$

Compute approximations to the roots of (*) to $O(\varepsilon^2)$

Then evaluate the estimate using $\varepsilon = 0.001$

To compare your estimates to the "exact" answer, factor

(*) as $(x-2)(x^2+ax+b) = 0$

i.e. one of the roots is exactly 2. The other 2 you can find by solving for the roots of

$$x^2+ax+b=0$$