

HW3

① $\dot{\underline{x}} = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \underline{x}$

Solve the above system. Note: while one of the eigenvalues is repeated, you will not have trouble find 2 associated vectors.

② Solve $\dot{\underline{x}} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \underline{x}$; $\underline{x}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

③ Optional: show that the eigenvalues of $T^{-1}AT$ are the same as those for A

④ Optional: verify the Cayley Hamilton Theorem for $A = \begin{pmatrix} 1 & -1 & -1 \\ 2 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$

⑤ Compute e^{At} , given $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{pmatrix}$

⑥ Solve $\dot{\underline{x}} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \underline{x} + \begin{pmatrix} \sin t \\ 0 \end{pmatrix}$ with $\underline{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

⑦ Optional: fill in the details for the problem done in class, namely

$$\dot{\vec{x}} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \vec{x} + \begin{pmatrix} 0 \\ 0 \\ e^t \cos 2t \end{pmatrix}; \quad \vec{x}(0) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

⑧ Solve $\dot{\vec{x}} = \begin{pmatrix} 3 & -4 \\ -1 & -1 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t$

$$\vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$