Exercises

1. In the blast wave problem discussed in class, assume instead $g(t, r, \rho, e, P) = 0$ where P is the ambient pressure.
   (a) How many dimensionless groups can we form?
   (b) Using these groups, does it still follow that r varies like the two-fifths power of t?

2. A physical system is described by a law $f(E, P, A) = 0$ where E is energy, P is pressure, A is area. Show that

$$PA^{3/2}/E = \text{constant, or } \quad E = kPA^{3/2}$$

where k is a constant.

3. We want to find the power P that must be applied to keep a ship of length ℓ moving at constant speed V. Assume P depends on the water density ρ, acceleration of gravity g,

E1/2

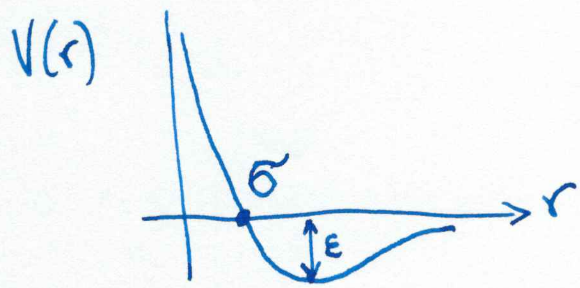
the viscosity of water ν (units of length squared/time) as well as ℓ and V. Show that

$$\frac{P}{\rho \ell^2 V^3} = f(Fr, Re)$$

where $Fr = \dfrac{V}{\sqrt{\ell g}}$ $\quad Re = \dfrac{V\ell}{\nu}$

4. The Lennard-Jones potential is defined by

$$V(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right], \quad \begin{matrix}\sigma \\ \varepsilon\end{matrix} \text{ are constants.}$$



This is the potential for many molecular systems, separated by a distance r. In general it is known that the viscosity μ (mass/length/time) of a gas depends only on the temperature & molecular properties.

Show that it is not possible, however, for

$$\mu = f(m, \varepsilon, \sigma, T)$$

for some function f, where m is the molecular mass, T is the temperature. On the other hand, by including Boltzmann's constant k $(1.38 \cdot 10^{-6}\ \mathrm{erg}/^\circ K)$ show that

$$\frac{\mu\sigma^2}{\sqrt{m\varepsilon}} = f\left(\frac{kT}{\varepsilon}\right)$$

5. Consider the heat equation problem on a 1D rod:

$$\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = 0$$

where $u = u(x,t)$ is temperature

$0 \leq x \leq L$ is the position coordinate

$0 < t$ is time

k is thermal diffusivity

k is in dimension L^2T^{-1}

the boundary conditions are

$u(x=0,t) = T_0$ for $t>0$

and $u(x=L,t) = T_0$ for $t > 0$

T_0 is a constant known temperature.

The rod is initial at $u(x,0) = 0$ for $0 < x < L$

(a) Find the scaling that turns the problem into

$$\frac{\partial \hat{u}(\hat{x},\hat{t})}{\partial \hat{t}} - \frac{\partial^2 \hat{u}(\hat{x},\hat{t})}{\partial \hat{x}^2} = 0 \qquad \hat{t} > 0, \quad 0 < \hat{x} < 1$$

$$\hat{u}(\hat{x},0) = 0 \qquad 0 < \hat{x} < 1$$

$$\hat{u}(0,\hat{t}) = \hat{u}(1,\hat{t}) = 1 \qquad \hat{t} > 0$$

(b) What can you say about the typical rate of change of u by your choice of time scale?

(c) ~~Find a functional form for u in terms of dimensionless groups.~~