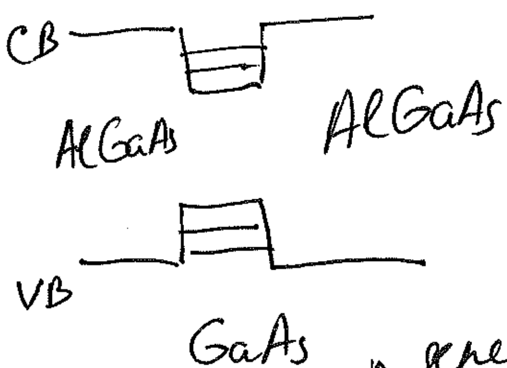
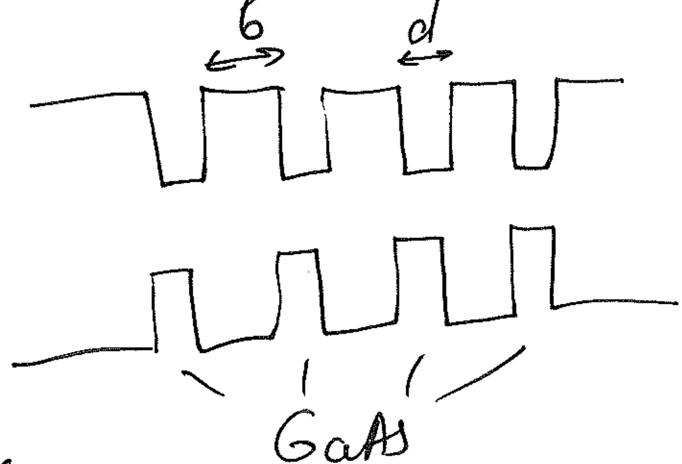
# Quantum - confined structures

Start from 2D ⇒ quantum wells (QW)

Example




QW ↑ generally need lattice-matched materials




MQW or superlattice
↑ large b ↑ small b ⇓ QW are coupled by tunneling

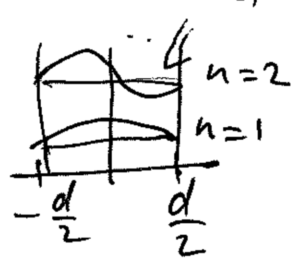
$$E = E_n + \frac{\hbar^2 \vec{k}^2}{2m^*}$$

↑ quantized

$$\Psi(x,y,z) = \underbrace{\Psi_k(x,y)}_{\frac{1}{\sqrt{A}} e^{i\vec{k}\cdot\vec{r}}} \Psi_n(z)$$

↖ if infinite walls ⇓

$$\Psi_n(z) = \sqrt{\frac{2}{d}} \sin\left(\frac{\pi n}{d} z + \frac{\pi n}{2}\right)$$




$$E_n = \frac{\hbar^2}{2m^*}\left(\frac{\pi n}{d}\right)^2$$

If $m^* = 0.1 m_e$

$d = 10\,nm$

⇒ $E_1 = 38\,meV$

$E_2 = 150\,meV$

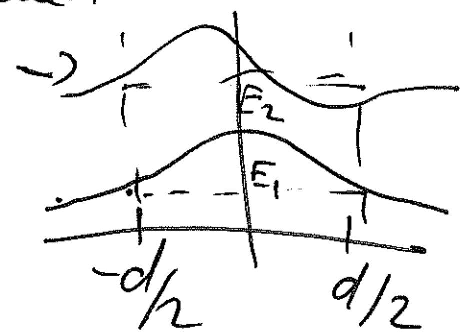
$> k_B T$ (25 meV at room $t°$)

⇒ observe QM effects

- since $E_n \sim \frac{1}{m^*}, \frac{1}{d^2}$ ⇒ low mass particles in narrow QW have highest energies
- e, hh, & lh will have different $E_n$ ⇒ in VB hh will typically dominate as they form ground-state level

In real QW ⇒ confinement is reduced

$d = 10\,nm$ GaAs / $Al_{0.3}Ga_{0.7}As$ QW →

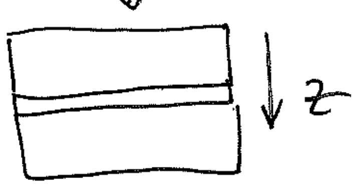



| Energies (meV) | finite well | infinite well |
|---|---|---|
| e1 | 32 | 57 |
| e2 | 120 | 227 |
| e3 | 247 | 550 |
| hh 1 | 7 | 11 |
| 2 | 30 | 44 |
| 3 | 66 | 100 |
| 4 | 112 | 177 |
| lh 1 | 21 | 40 |
| 2 | 78 | 160 |

⇒ infinite well model overestimates quantiz.

## QW absorption

$\hbar\omega$




transition rate; position of $e^-$; light wave E-field

$$W_{i\to f} = \frac{2\pi}{\hbar}\,\left|\langle f|\,-e\vec{r}\cdot\vec{\mathcal{E}}\,|i\rangle\right|^2 g(\hbar\omega)$$

electric dipole approx. DOS

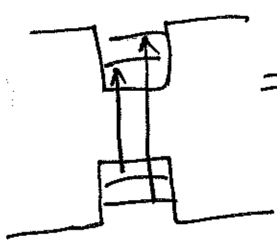
justified since $|\vec{k}_{photon}| \ll |\vec{k}_{electron}|$

⇓

$\vec{\mathcal{E}}$ is in the x-y plane

⇓ e.g.

$$M = \langle f|x|i\rangle = \int \psi_f^*(\vec{r})\,x\,\psi_i(\vec{r})\,d^3\vec{r}$$



$$\Rightarrow \psi_i \equiv |i\rangle = \frac{1}{\sqrt{A}}\,u_v(\vec{r})\,\varphi_{hn}(z)\,e^{i\vec{k}_{xy}\cdot\vec{r}_{xy}}$$

$$\psi_f = \frac{1}{\sqrt{A}}\,u_c(\vec{r})\,\varphi_{en'}(z)\,e^{i\vec{k}'_{xy}\cdot\vec{r}_{xy}}$$

Since $|\vec{K}_{photon}| << |\vec{K}_{electron}| \Rightarrow \vec{K}_{xy} \approx \vec{K}'_{xy}$ (vertical transition, just like in 3D)

$$M = \underbrace{M_{cv}}_{\langle u_c|x|u_v\rangle} \underbrace{M_{nn'}}_{\text{e-h overlap}} = \langle en'|hn\rangle = \int_{-\infty}^{+\infty} \varphi^*_{en'}(z)\,\varphi_{hn}(z)\,dz$$

$$\langle u_c|x|u_v\rangle = \int u_c^*(\vec{r})\, x\, u_v(\vec{r})\, d^3\vec{r}$$

Valence-cond. band dipole moment

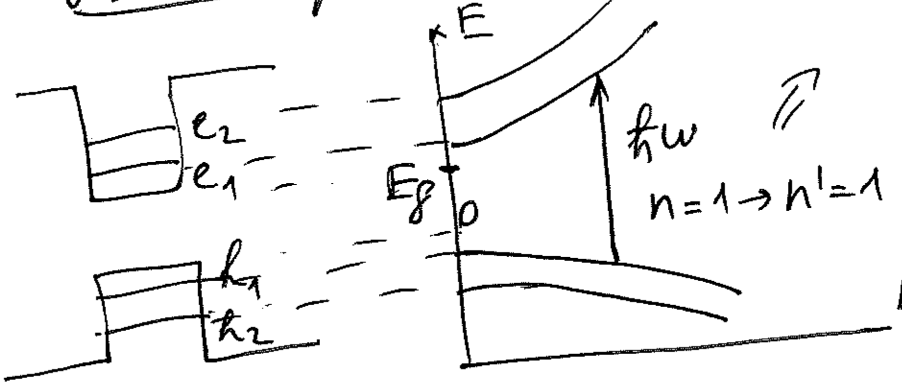
$M_{cv}$ is typically non-zero & large ← e.g. strongly allowed electric-dipole transitions in GaAs

$$M_{nn'} = \frac{2}{d}\int_{-d/2}^{d/2} \sin\left(\frac{\pi n}{d}z + \frac{\pi n}{2}\right)\sin\left(\frac{\pi n'}{d}z + \frac{\pi n'}{2}\right)dz = \delta_{n'n}$$

$\Downarrow$

$\boxed{\Delta n = 0}$ ← selection rule

<u>2D absorption</u>

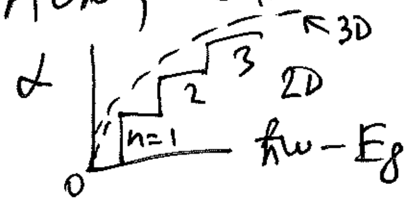



$$\hbar\omega = E_g + E_{hh_1} + \frac{\hbar^2 k_{xy}^2}{2m^*_{hh}} + E_{e_1} + \frac{\hbar^2 k_{xy}^2}{2m^*_e} = E_g + E_{hh_1} + E_{e_1} + \frac{\hbar^2 k_{xy}^2}{2\mu}$$

$\mu$ ← reduced mass

So, the absorption threshold shifts to $E_g + E_{hh_1} + E_{e_1}$.

Also, $\vec{k}$ is now in 2D $\Rightarrow$ joint DOS $g_{2D}(E) = \frac{\mu}{\pi\hbar^2} \Rightarrow$



Threshold energy for the nth transition $\Rightarrow$

$$\hbar\omega = E_g + \frac{\hbar^2\pi^2 n^2}{2m_e^* d^2} + \frac{\hbar^2\pi^2 n^2}{2m_h^* d^2} = E_g + \frac{\hbar^2\pi^2 n^2}{2\mu d^2}$$

<u>Example</u>

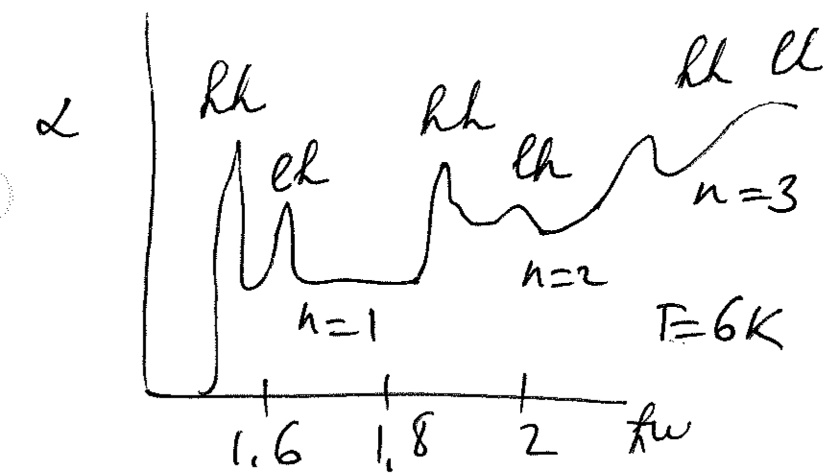
20 nm GaAs QW vs bulk GaAs @ 300 K

approximate with an infinite well model $\Rightarrow$ $E_{e_1} = 14\,meV$

$E_g = 1.424\,eV \Rightarrow E_g + E_{hh_1} + E_e = 1.44\,eV$ $\quad E_{hh_1} = 2\,meV$

bulk $\qquad$ QW $\qquad \Rightarrow$ blue shift of $\sim 10\,nm$

GaAs MQW




$\swarrow$ steps due to 2D confinement

peaks are excitonic

Note: excitons in 2D are more bound $\Rightarrow$ GaAs MQW 10 nm

$\Downarrow$

excitonic effects in QW can be observed at room t° $\qquad$ compare to 4.2 meV in bulk GaAs $\rightarrow$ $E_{bind} = 10\,meV$