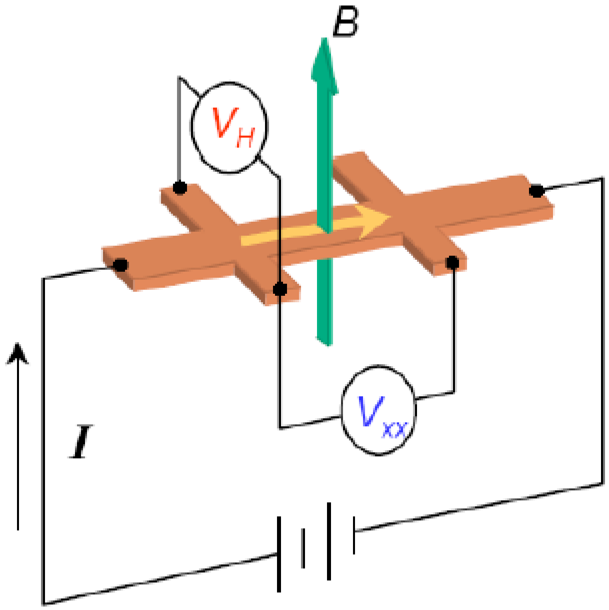
Low-dimensional semiconductors. Magnetic field effects.

PH 673

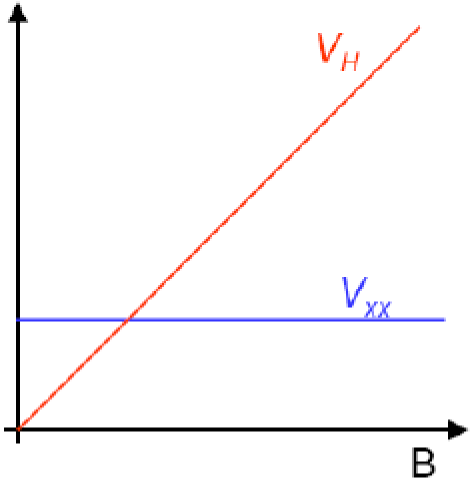
Nanoscience and nanotechnology

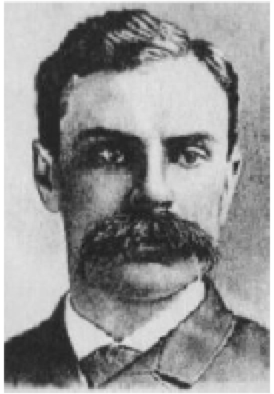
October 3, 2025

Hall effect



Ordinary Hall effect



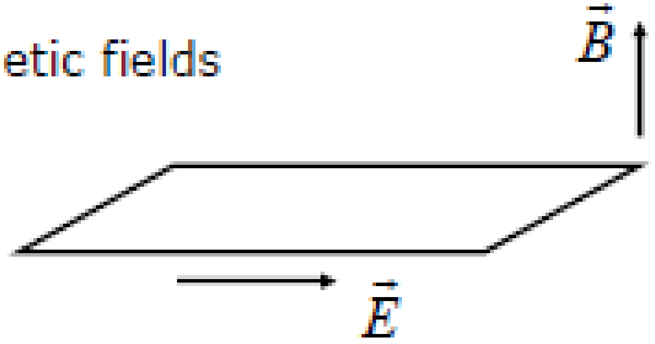


Ed Hall, 1879

$$R_H = V_H / I = B/ne$$

Hall effect

Electron moving in crossed electric and magnetic fields



Lorentz force: $\boxed{\vec{F} = e\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)}$

y-direction: no force $\boxed{E_y = \frac{1}{c}Bv_x}$ $\Rightarrow$ voltage in the transverse direction Hall effect

$$\boxed{\begin{matrix} I = env_x W \\ V_H = E_y W \end{matrix} \Rightarrow R_H = \frac{V_H}{I} = \frac{B}{enc}}$$ - (classical) Hall resistance

Impurity independent $\omega_c \gg 1/\tau$

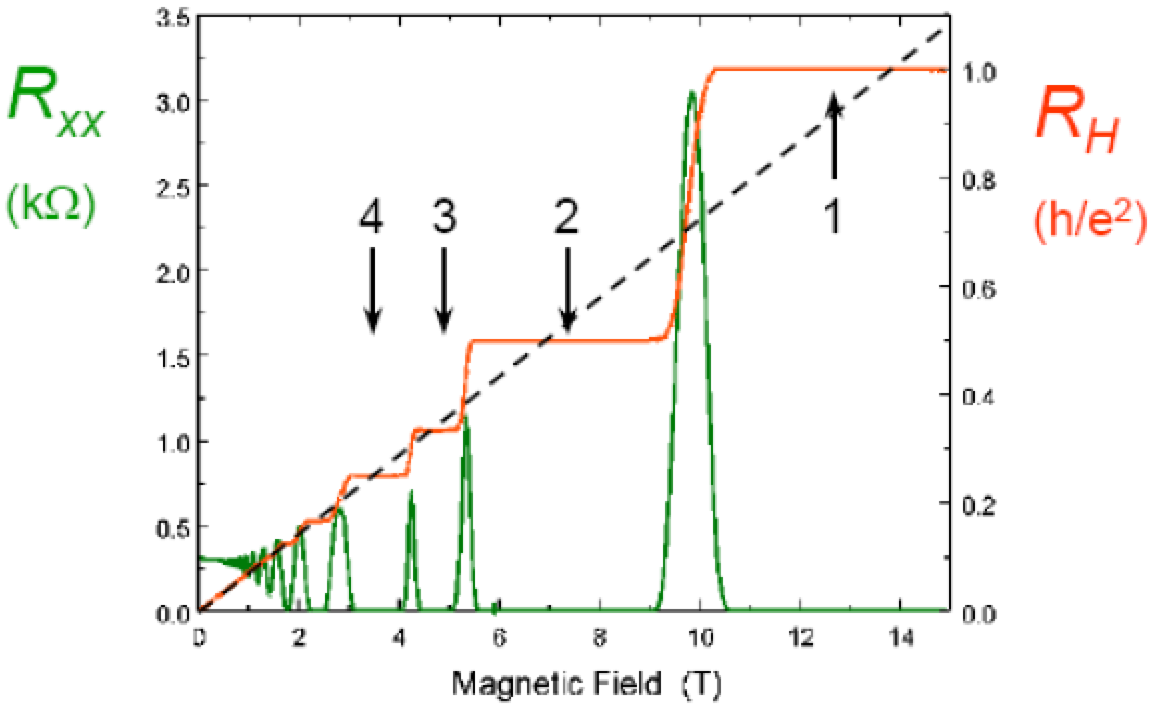
Proportional to the magnetic field

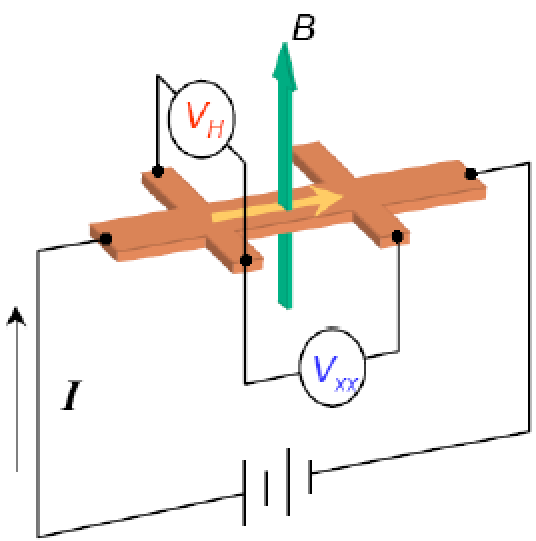
the quantum Hall effect in a 2DEG

1985

Si-MOSFET: two-dimensional device

Klaus von Klitzing





$$R_{Hall} = \frac{h/e^2}{j} = 25812.807 \text{ Ohms} / \text{integer}$$

Hall resistance (R_{xy}) increases in step-wise way to well defined quantized values).
At these well-defined values: $R_{xx} = 0$!!!

Landau quantization in 2D

A free electron in magnetic field: $\vec{B} \parallel \hat{z}$

Schrödinger equation: $$-\frac{\hbar^2}{2m}\left(\vec{\nabla} + \frac{ie\vec{A}}{\hbar c}\right)^2 \psi = \varepsilon\psi$$ $$A_y = Bx;\ A_x = A_z = 0$$

Solutions: labeled by one index *n*

$$\psi_n(\vec{r}) = \exp(ik_y y)\varphi_n(x - \hbar c k_y / eB)$$

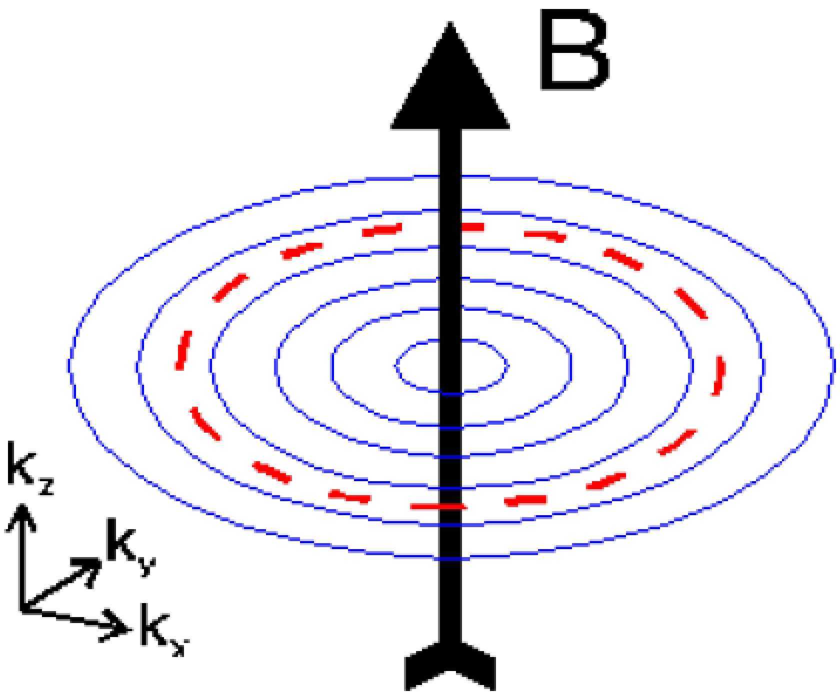
φ_n - wave functions of a harmonic oscillator

Energies: $\varepsilon_n = (e\hbar B / mc)(n + 1/2) \equiv \hbar\omega_c(n + 1/2)$ - strongly degenerate!!

(Landau levels)

$\omega_c = eB / mc$ - cyclotron frequency

Electrons in strong magnetic fields

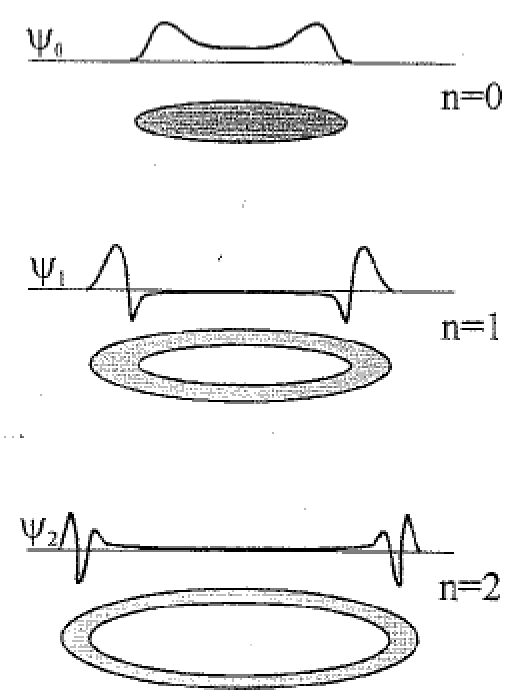


The kinetic energy in a magnetiuc field, B, is quantized in Landau levels due to quantized cyclotron motion.

A quantum mechanical calculations shows that the energy levels are given by:

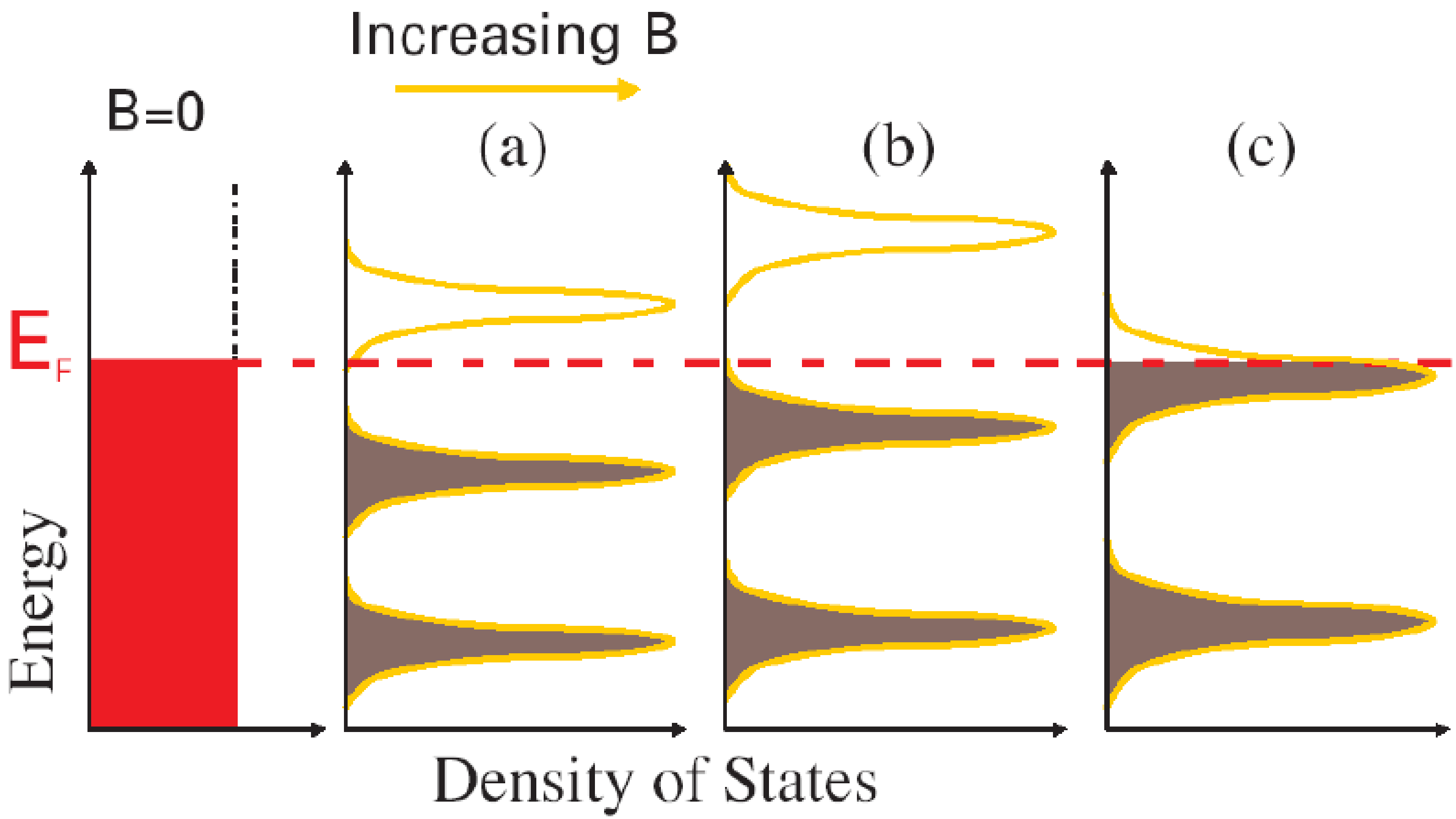
$$E_n = (n + 1/2)\ \hbar\omega_c$$

$$\omega_c = eB/m$$



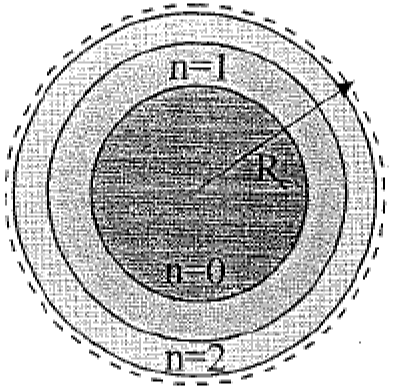
Recall classical argument to calculate $\hbar\omega_c$

Density of states in the magnetic field



Peak broadening is caused by disorder

Filling factor



At $B \neq 0$, all zero-field states within a range $\hbar\omega_c$ are condensed into a single Landau level. The number of states per level, per unit area is therefore:

$$N_L = D_{2D}(E)\, \hbar\omega_c = eB/h = \Phi/\Phi_0 S$$

(This is also the number of flux quanta threading unit area. This is not a coincidence; the number of different electron wave functions that can fits in each LL is equal to the number flux quanta. S = sample area)

Thus, the larger B the more states fit into a Landau level !!!!!

When particles are put into these states, the *filling factor* ν is equal to the number of Landau levels filled. Clearly for partially filled LLs ν is a fraction. As the magnetic field is increased the filling factor decreases since more particles can be put in each LL; i.e., the number of filled Landau levels decreases and at some point even the last Landau level will be depopulated (what is a typical value of the magnetic field for that ??).

filling factor = areal density of particles/areal density of flux quanta = $n/N_L = nh/(eB)$

Filling factor

$\boxed{\varepsilon_n = \hbar\omega_c(n+1/2)}$ How many states are there in the area *A* on one LL?

$$\boxed{\psi_n(\vec{r}) = \exp(ik_y y)\varphi_n(x - \hbar c k_y / eB)}$$

Quantize k_y $\boxed{k_y = 2\pi m_y / W}$

Condition for the maximal *x*: $\boxed{L = \hbar c k_y / eB = k_y l^2 = 2\pi l^2 m_{\max} / W}$

$\boxed{l = \sqrt{\hbar c / eB}}$ - magnetic lenth

# of states: $\boxed{m_{\max} = A / 2\pi l^2 \equiv n_B A}$

Filling factor: $$\boxed{\nu = \frac{\#electrons}{\#states} = \frac{n}{n_B} = \frac{2\pi\hbar c n}{eB}}$$

Fix electron concentration: Higher magnetic fields correspond to lower filling factors

Integer Quantum Hall effect

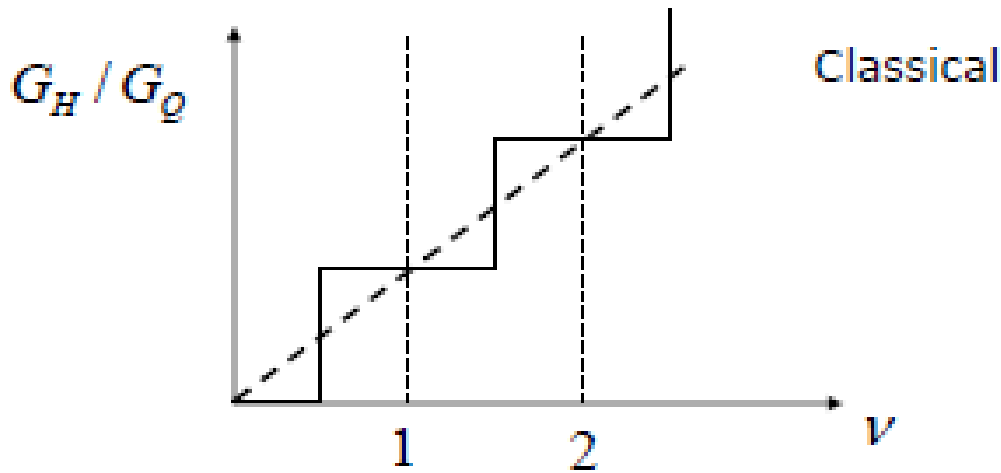
$$G_H = \frac{enc}{B}$$

Integer filling factor: $\nu = m \in \mathbb{N}$

$$G_H = G_Q m$$

$$G_Q \equiv e^2 / 2\pi\hbar$$

Experiment (Von Klitzing '80)

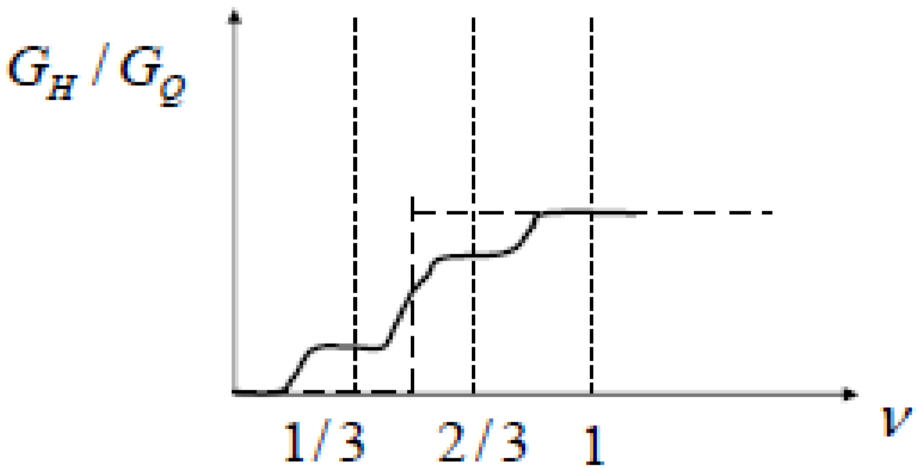


At plateaus: no longitudinal resistance

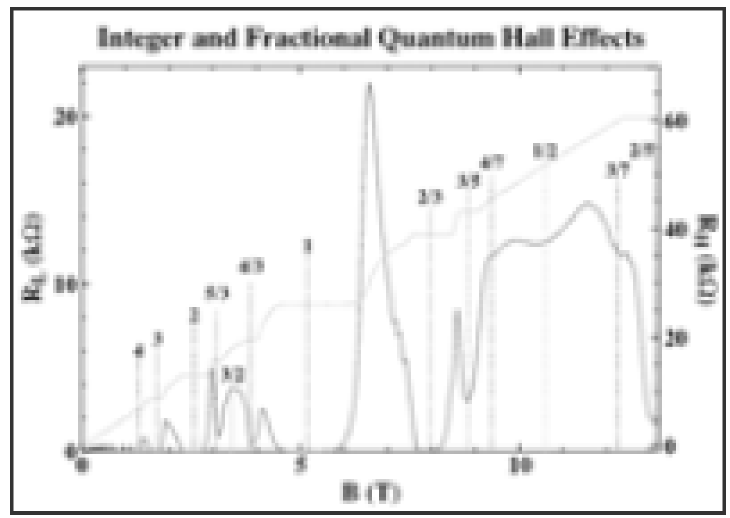
Fractional Quantum Hall effect

Experiment (Tsui, Störmer, Gossard '81) : additional plateaus at

$$\nu = p/(2m+1)$$



$$\boxed{G_H = \frac{p}{2m+1} G_Q}$$



New states of matter (Laughlin states)

Interactions are important!

Laughlin wave function

Consider a state with $\nu = 1/(2m+1)$

Wave function: $z_n = x_n + iy_n$

$$\boxed{\Psi(z_1 \ldots z_n) \propto \prod_{ij} (z_i - z_j)^{(2m+1)} \prod_i e^{-|z_i|^2/2l}}$$

Fermionic!

<u>Properties:</u>

- Equivalent to a system of Coulomb charges in a confining potential;
- Excitations: fractional charge *e/(2m+1)*, can be probed in experiments
- Excitations: fractional statistics (not bosonic nor fermionic)
- Edge states: *(2m+1)* channels; not very well understood

$\nu = p/(2m+1)$ Excitations still have fractional charge *e/(2m+1)* and fractional statistics

Tsui, Stormer, Laughlin: Nobel Prize in Physics 1998

Observation of an Even-Denominator Quantum Number in the Fractional Quantum Hall Effect

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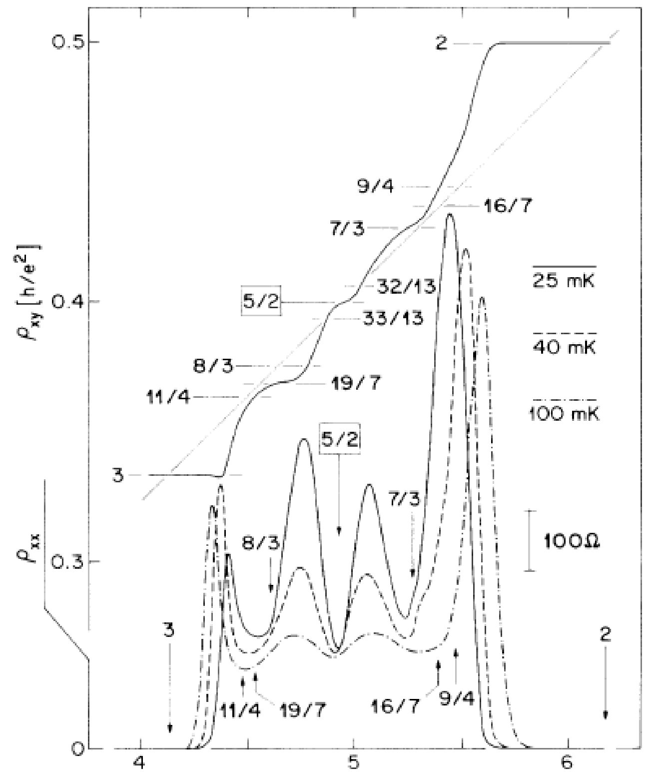

An even-denominator rational quantum number has been observed in the Hall resistance of a two-dimensional electron system. At partial filling of the second Landau level $\nu=2+\frac{1}{2}=\frac{5}{2}$ and at temperatures below 100 mK, a fractional Hall plateau develops at $\rho_{xy}=(h/e^2)/\frac{5}{2}$ defined to better than 0.5%. Equivalent even-denominator quantization is absent in the lowest Landau level under comparable conditions.

Direct observation of a fractional charge

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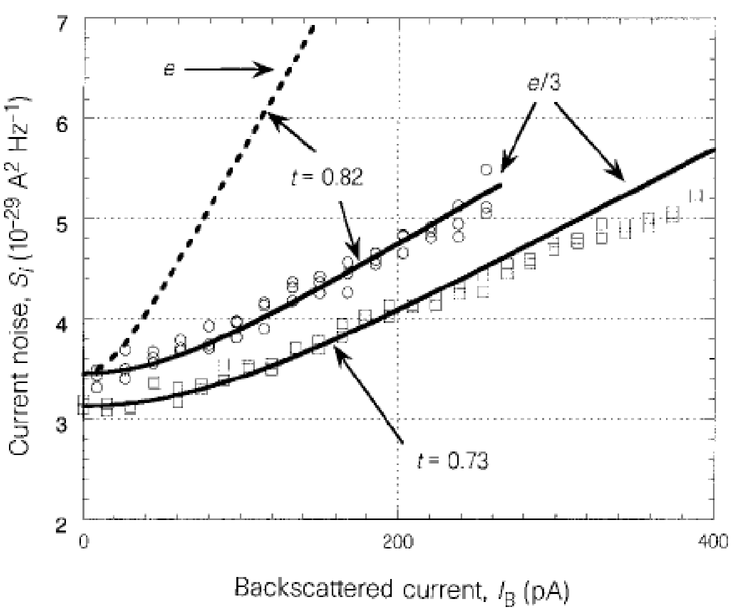

Since Millikan's famous oil-drop experiments[1], it has been well known that electrical charge is quantized in units of the charge of an electron, *e*. For this reason, the theoretical prediction[2,3] by Laughlin of the existence of fractionally charged 'quasiparticles'—proposed as an explanation for the fractional quantum Hall (FQH) effect—is very counterintuitive. The FQH effect is a phenomenon observed in the conduction properties of a two-dimensional electron gas subjected to a strong perpendicular magnetic field. This effect results from the strong interaction between electrons, brought about by the magnetic field, giving rise to the aforementioned fractionally charged quasiparticles which carry the current. Here we report the direct observation of these counterintuitive entities by using measurements of quantum shot noise. Quantum shot noise results from the discreteness of the current-carrying charges and so is proportional to both the charge of the quasiparticles and the average current. Our measurements of quantum shot noise show unambiguously that current in a two-dimensional electron gas in the FQH regime is carried by fractional charges—*e*/3 in the present case—in agreement with Laughlin's prediction.

Selected for a Viewpoint in *Physics*
PRL **106,** 236804 (2011) PHYSICAL REVIEW LETTERS week ending 10 JUNE 2011

Fractional Quantum Hall States at Zero Magnetic Field

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High-Temperature Fractional Quantum Hall States

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Nearly Flatbands with Nontrivial Topology

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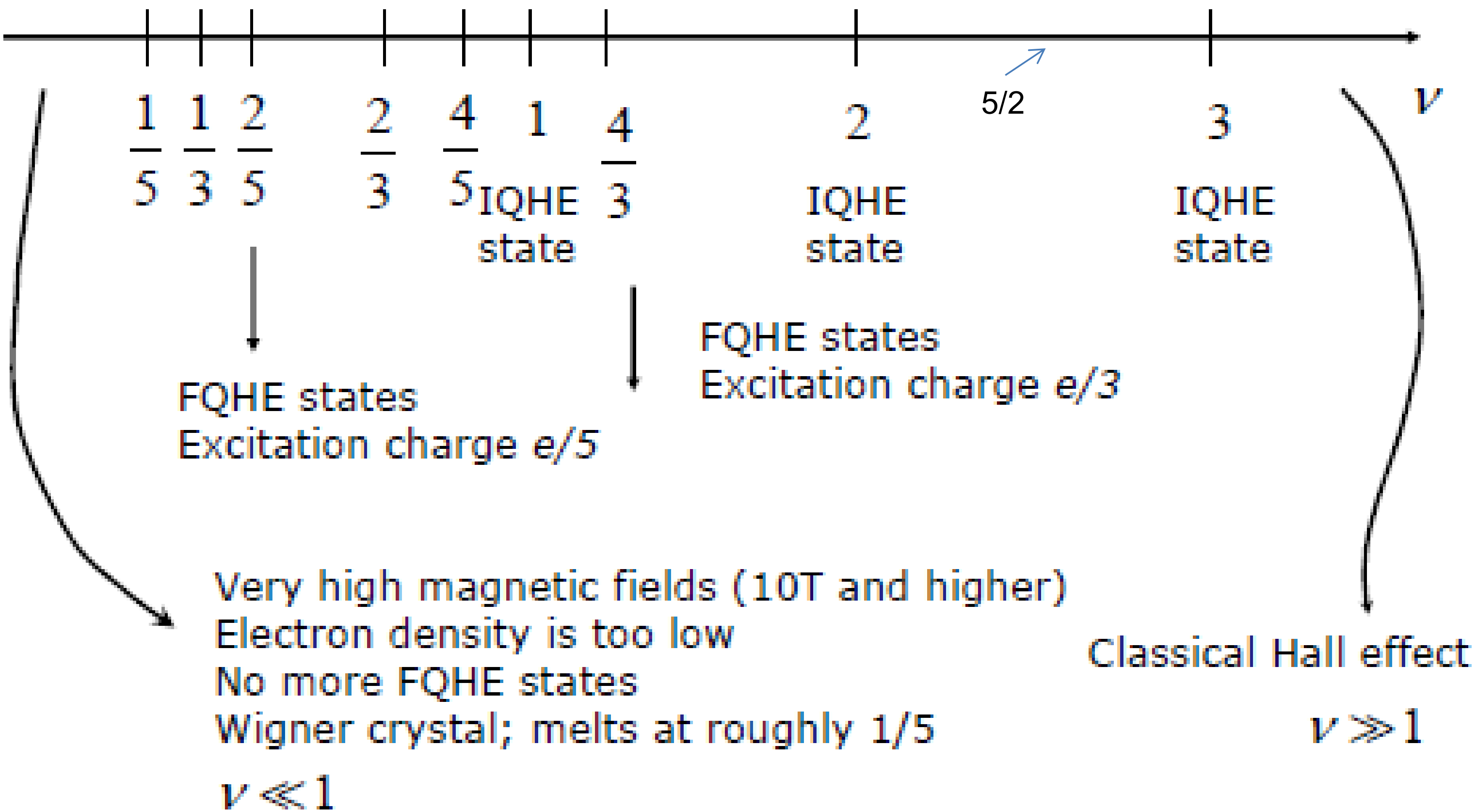
Electrons in magnetic field



Jain: composite fermions (Wolf Prize in Physics 2025)

Wigner crystal

Consider electrons at low density

Intuition from classical physics:

Low density $\Rightarrow$ Long distances between electrons $\Rightarrow$ Weak interactions

What do we get from quantum mechanics?

Kinetic energy: $E_{kin} \sim p^2 / 2m \sim \hbar^2 / ma^2$

Potential energy (no screening): $E_{pot} \sim e^2 / a$

Low density of electrons: $\boxed{a > \hbar^2 / me^2 = a_B}$

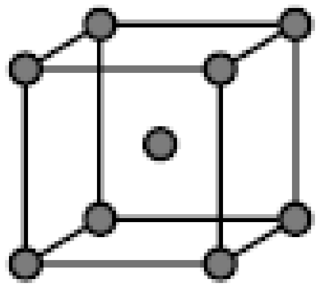
Potential energy dominates!

Wigner crystal

What is the ground state of the electron system at low density?

We need to minimize the potential energy!

Wigner crystal   Bulk-centered cubic lattice



$$a > \hbar^2 / me^2 = a_B$$

Problems with observation:

- Difficult to create a low density
- No uniform positive background
- Destroyed by disorder

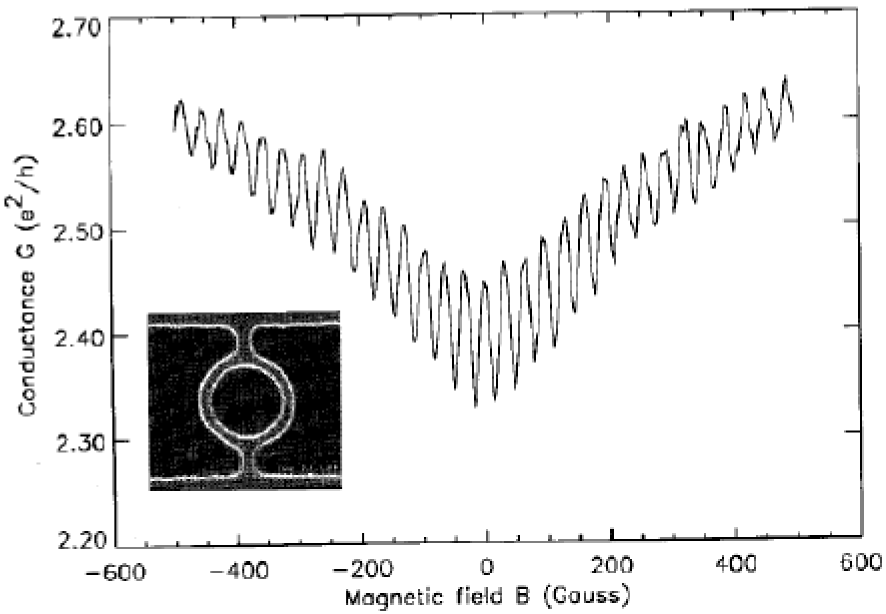
Realisations:

- Electrons at helium surface
- Semiconducting heterostructures

Phase coherence

at low temperatures

$$\ell_\phi \geq 100\ \mu\text{m}$$



The resistance of a small ring with a diameter of about 1 micron (the light gray areas in the inset) as a function of a magnetic field applied perpendicular to the ring plane shows periodic oscillations, known as *Aharonov-Bohm oscillations*.

They indicate that a significant fraction of the electrons traverse the ring phase coherently.