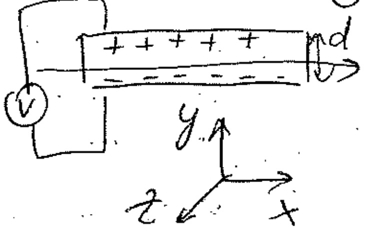
PHYS 671 Nano

Lecture # 3

Transport in magnetic fields

Recall Hall effect (1879):



metals ↓ Hall field

$$\tilde{R}_H = \frac{E_y}{j_x B} = -\frac{1}{ne(c)}$$

measure voltage V_H, $E_y = \frac{V_H}{d}$

Hall const; current density; in SI; n ← carrier density

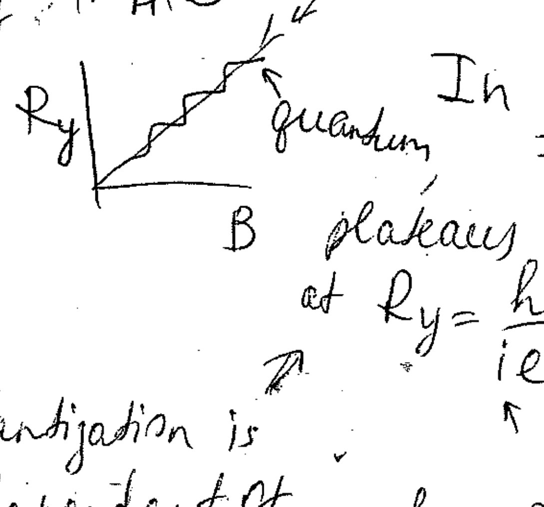
Hall effect is used for measurements of n, B, sign of the majority carrier, its mobility, etc.

In semiconductors, $\tilde{R}_H = \frac{\mu_h^2 p - n\mu_e^2}{e(n\mu_e + p\mu_h)^2}$

electron & hole concentration
& hole mobility

Hall resistance
$R_y = |\tilde{R}_H| B$

So



plateaus at $R_y = \frac{h}{ie^2}$

$i = 1, 2, 3, \ldots$

In 2D structures ⇒ quantum Hall effect ⇒ Hall field is quantized

von Klitzing (1980)

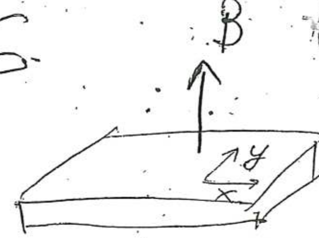
Nobel prize 1985

quantization is independent of the material

$\frac{h}{e^2} = R_K = 25.813\,k\Omega$ ⇒ used as Ohm standard

of free-space permeabil. $\frac{\mu_0 c}{2\alpha}$ ← fine-struct const; von Klitzing const

Consider $\vec{B} \parallel Oz$; 2DEG



$\vec{\nabla} \times \vec{A} = \vec{B} \Rightarrow$ choose Landau gauge $\Rightarrow A_x = A_z = 0$

$A_y = Bx$

Consider motion in x–y plane: $\underbrace{p_y}_{-i\hbar\frac{\partial}{\partial y}} \to p_y - e\overbrace{A_y}^{Bx}$

$$\left[\frac{p_x^2}{2m^*} + \frac{(p_y - eA_y)^2}{2m^*}\right]\Psi(x,y) = E\Psi(x,y)$$

$$-\frac{\hbar^2}{2m^*}\underbrace{\nabla^2}_{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}}\Psi + \left[\frac{i\hbar eBx}{2m^*}\frac{\partial}{\partial y} + \frac{e^2B^2}{2m}x^2\right]\Psi(x,y) = E\Psi$$

Look for $\Psi(x,y) = e^{ik_y y}\,\Psi(x) \Rightarrow$

$$-\frac{\hbar^2}{2m^*}\Psi''(x) + \frac{\hbar^2}{2m^*}k_y^2\,\Psi(x) - \frac{\hbar k_y eB}{2m^*}x\,\Psi(x) +$$

$$+\frac{e^2B^2}{2m^*}x^2\,\Psi(x) = E\Psi(x) \Rightarrow$$

$$-\frac{\hbar^2}{2m^*}\Psi''(x) + (\hbar k_y - eBx)^2\frac{1}{2m^*}\Psi(x) = E\Psi(x)$$

harmonic oscillator potential!

$$\Psi_n(x) \sim e^{-\frac{(x-x_0)^2}{2l_m^2}} H_n\left(\frac{x-x_0}{l_m}\right),$$

$$l_m = \sqrt{\frac{\hbar}{eB}} \; ; \quad x_0 = \frac{\hbar k_y}{eB} \; ; \quad \omega_c = \frac{eB}{m^*}$$

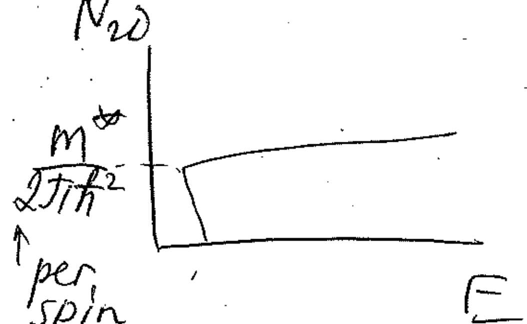
magnetic length ($B = 1T \Rightarrow l_m \simeq 25\,nm$)

cyclotron frequency

$$E_n = \hbar\omega_c \left(n + \frac{1}{2}\right)$$

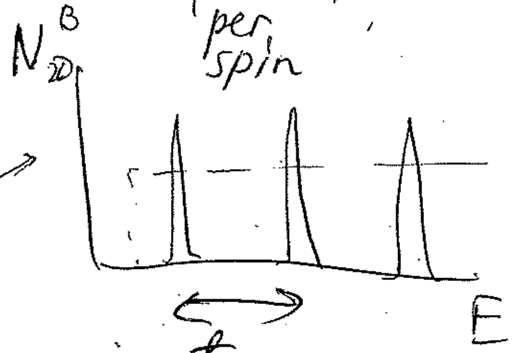
Landau levels

So, $B = 0 \Rightarrow$



$B \neq 0 \Rightarrow$

$\frac{1}{\hbar\omega_c}$



need;
1) $k_B T << \hbar\omega_c$
2) $l_f >> l_m$

How many states per LL? $\Rightarrow$

$$\frac{N_{2D}}{N_{2D}^B} = \frac{m^*}{2\pi\hbar^2}\hbar\omega_c = \frac{eB}{\cancel{m^*}}\frac{\cancel{m^*}}{2\pi\hbar} = \frac{eB}{h}$$

Hall constant

charge density (assuming i levels occupied) $\Rightarrow n = i \cdot \frac{eB}{h} \Rightarrow |\tilde{R}_H| = \frac{1}{ne} = \frac{h}{e^2 B i}$

Hall resistance $R_H = |\tilde{R}_H| B = \frac{h}{e^2 i}$

$i = 1, 2, 3, \ldots$

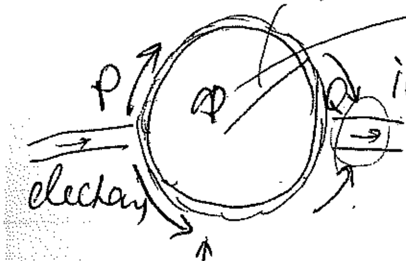
integer

<u>fill factor</u> $\Rightarrow \nu = \frac{n (\#\text{electrons})}{eB/h \; (\#\text{states})} = i$ in IQHE

The Aharonov – Bohm effect

1959: electron wave in a solid has a phase factor which can be controlled by a magnetic field

400nm magnetic flux (solenoid)



interference

electron momentum $\vec{p} + e\vec{A}$

metal wire ~50 nm thick

Change in phase as electron travels from P to Q:

$$\varphi(\vec{r}) = \frac{e}{\hbar}\int_P^Q \vec{A}\cdot d\vec{s}$$

along the path

Phase difference between upper and lower paths:

$$\Delta\varphi = \varphi_1 - \varphi_2 = \frac{e}{\hbar}\Big[\underbrace{\int}_{\text{lower}} \vec{A}\cdot d\vec{s} - \underbrace{\int}_{\text{upper}} \vec{A}\cdot d\vec{s}\Big] = \frac{e}{\hbar}\underbrace{\oint}_{\text{circle}} \vec{A}\cdot d\vec{s} =$$

since electron waves move in opposite direc with respect to $\vec{A}$

$$= \frac{e}{\hbar}\Phi, \qquad \oint \vec{A}\cdot d\vec{s}$$

Stoke's theorem

where $\Phi = \int_{\text{area}} (\vec{\nabla}\times\vec{A})\cdot d\vec{S} = \int_{\text{circle area}} \vec{B}\cdot d\vec{S}$

If $\Phi_0 = \frac{h}{e}$ – the quantum of flux $\Rightarrow \Delta\varphi = \frac{2\pi\Phi}{\Phi_0}$

Intensity of wave interference $\Rightarrow$

$$I \sim |\Psi_1 + \Psi_2|^2 \sim \cos(\Delta \varphi) = \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)$$

wave functions $\Psi_i \sim e^{i\varphi_i}$

if Φ is varied, interference effects are expected

So, change $\vec{B}$ and observe oscillations in conductance