Low-dimensional semiconductors. Transport properties.

PH 673

Nanoscience and nanotechnology

October 1, 2025

Useful resource: lecture notes from Delft University (the Netherlands):

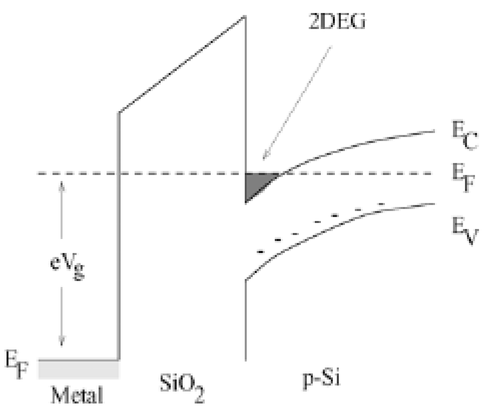
Semester-long courses in advanced solid state and in mesoscopic physics (mostly semiconductor nanostructures)

https://ocw.tudelft.nl/courses/mesoscopic-physics/

https://ocw.tudelft.nl/courses/advanced-solid-state-physics/

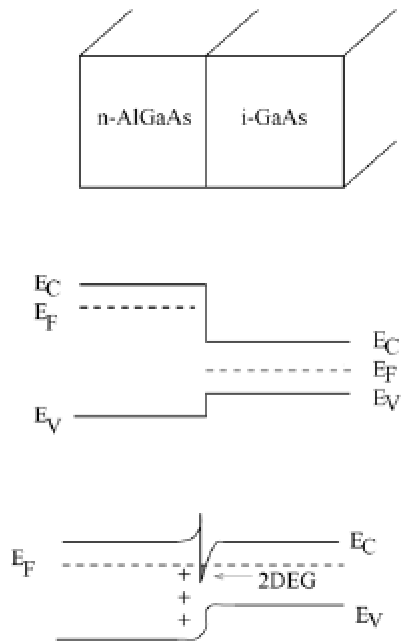
2D electron gas (2DEG)

band gap engineering



Metal-Oxide-Semiconductor (MOS) structures

2DEG is formed at the semiconductor-insulator interface



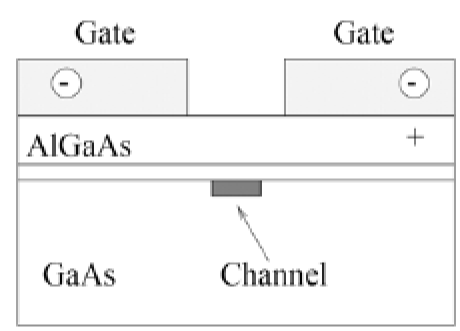
semiconductor heterostructure

2DEG is formed at the interface between two semiconductors

2DEG is a generic object for new physics

Nobel Prizes 1985, 1998, 2000

It serves as a building block for electronic devices



gates define device structures

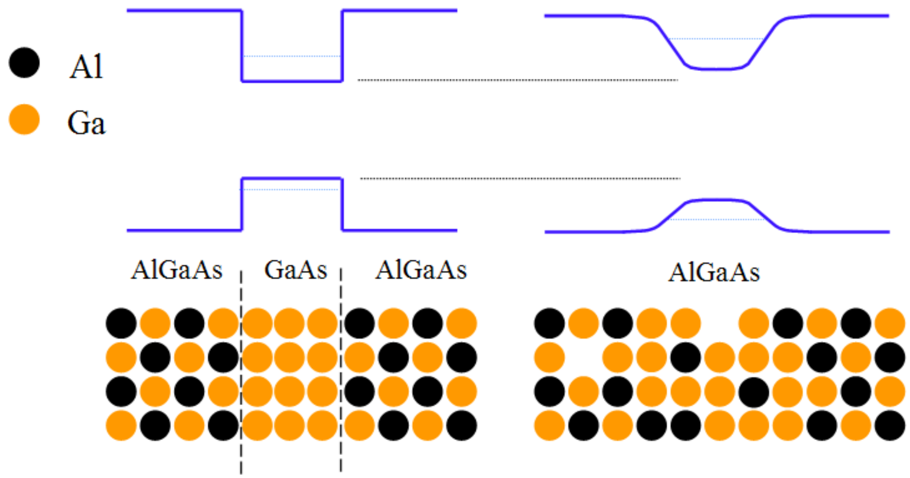
Nobel Prizes related to 2DEG

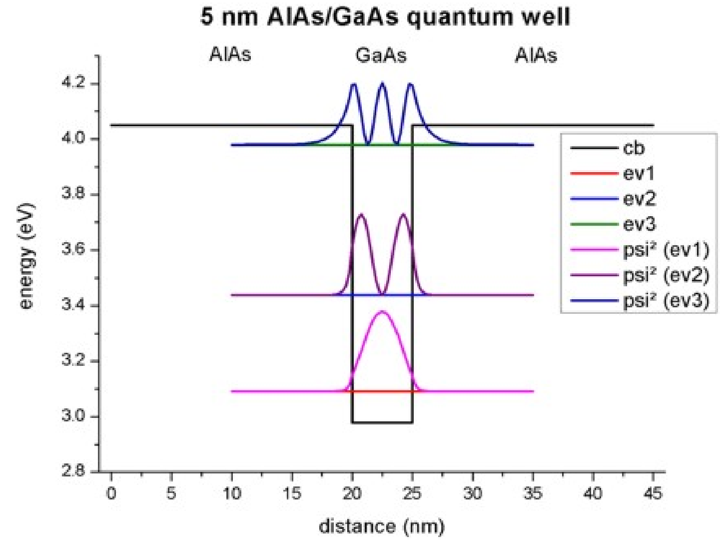
The electron transport in confined geometries is of high principal interest. Using a strong magnetic field applied perpendicular to a 2DEG, K. von Klitzing discovered the quantum Hall effect (Nobel Prize 1985) in samples supplied by M. Pepper and G. Dorda. Using even higher fields, D.C. Tsui and H.L. Stoermer (Nobel Prize 1998) discovered the fractional quantum Hall effect in ultrapure MBE material made by A.C. Gossard.

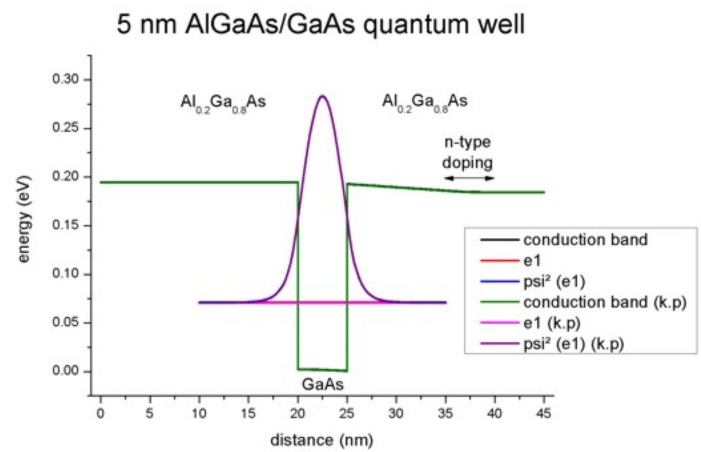
The Royal Swedish Academy of Sciences has awarded the Nobel Prize 2000 in Physics
“for basic work on information and communication technology”
The prize is being awarded with one half jointly to
Zhores I. Alferov, A.F. Ioffe Physico-Technical Institute, St. Petersburg, Russia, and
Herbert Kroemer, University of California at Santa Barbara, California, USA,
”for developing semiconductor heterostructures used in high-speed- and optoelectronics”
and one half to
Jack S. Kilby, Texas Instruments, Dallas, Texas, USA
”for his part in the invention of the integrated circuit”

http://nobelprize.org/

Quantum wells







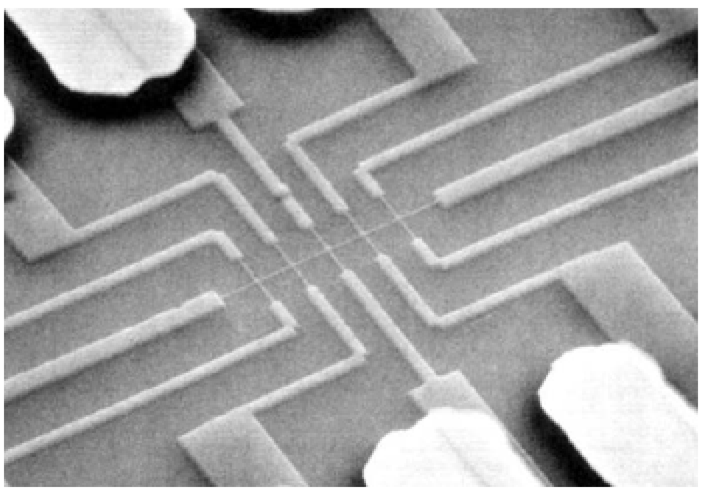
http://www.nextnano.de/nextnano3/tutorial/2Dtutorial4.htm

Quantum wires

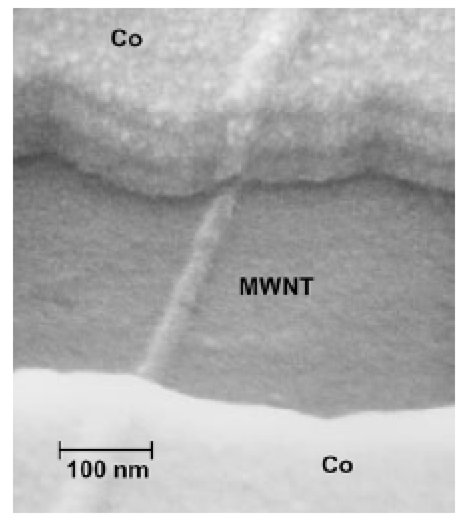
QUANTUM WIRES: free electron motion is restricted to **ONE** dimension **(1D)**

⇒ These **QUASI-ONE-DIMENSIONAL** structures may be realized using a variety of techniques

⇒ They also occur quite **NATURALLY** and examples of such structures include **CARBON NANOTUBES** and long **MOLECULAR CHAINS**



75-nm WIDE ETCHED QUANTUM WIRE



CARBON NANOTUBE BRIDGING TWO COBALT CONTACTS

M. L. Roukes *et al.*
Phys. Rev. Lett. 59, 3011 (1987)

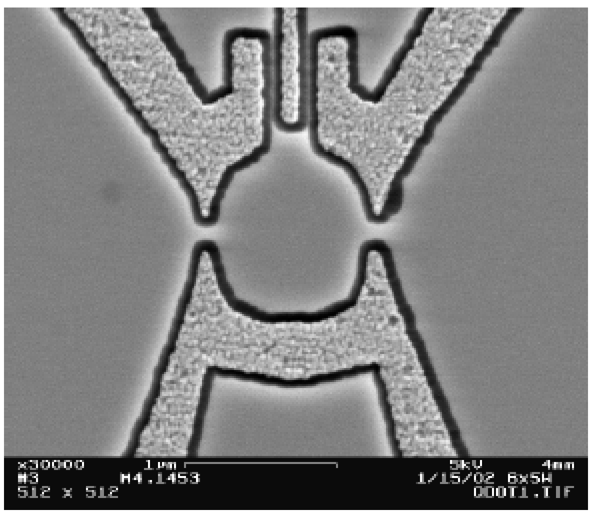
K. Tsukagoshi *et al.*
Nature 401, 572 (1999)

Quantum dots

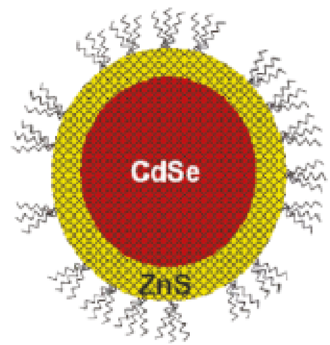
QUANTUM DOTS are structures (called zero-dimensional, **0-D**) in which electron motion is strongly confined in **ALL THREE** dimensions **(QUANTIZATION)** so that these structures may be viewed as **ARTIFICIAL ATOMS**

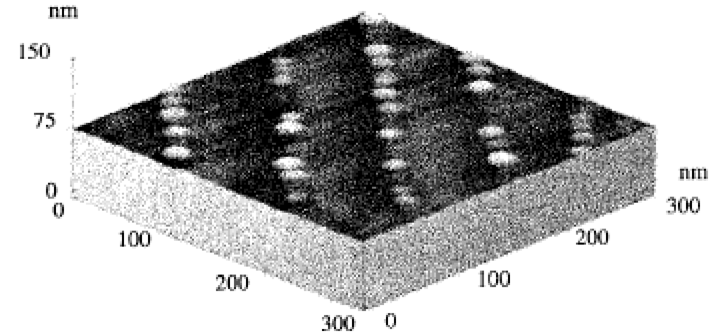
* These structures may be realized by a variety of different techniques and in a range of different materials

* Dominant transport mechanism: **SINGLE-ELECTRON TUNNELING**

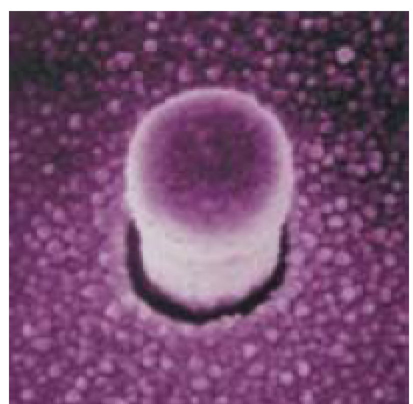


SEM IMAGE OF A GaAs/AlGaAs QUANTUM DOT REALIZED BY THE SPLIT-GATE METHOD

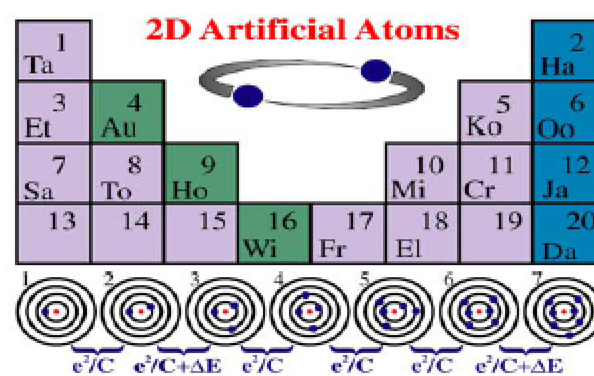




AFM IMAGE OF SELF-ASSEMBLED InGaAs QUANTUM DOTS



VERTICAL QUANTUM DOTS (DELFT, KOUWENHOVEN)

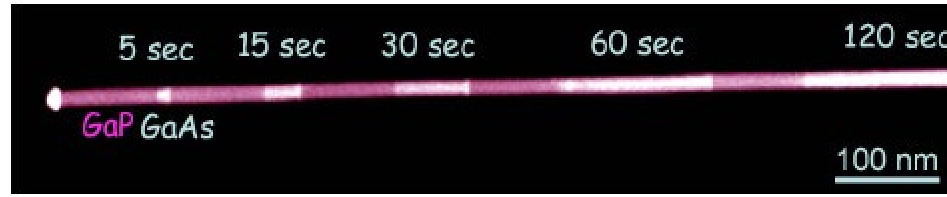


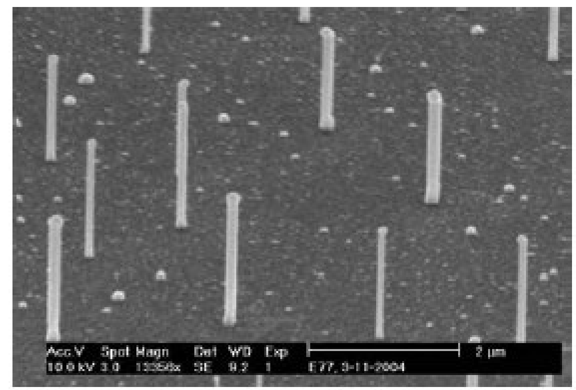
Example: II-VI , IV-VI, III-V semicond. - CdS/ZnS, CdSe/ZnS, CdSe/CdS, PbS, InAs/CdSe

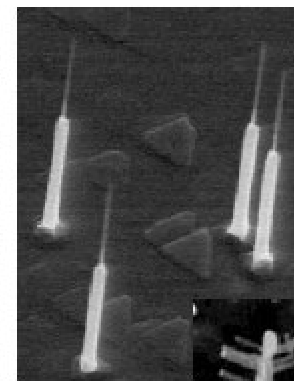
Nanoscale objects

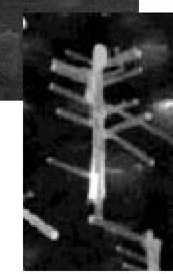
(organic) molecules

inorganic nanowires

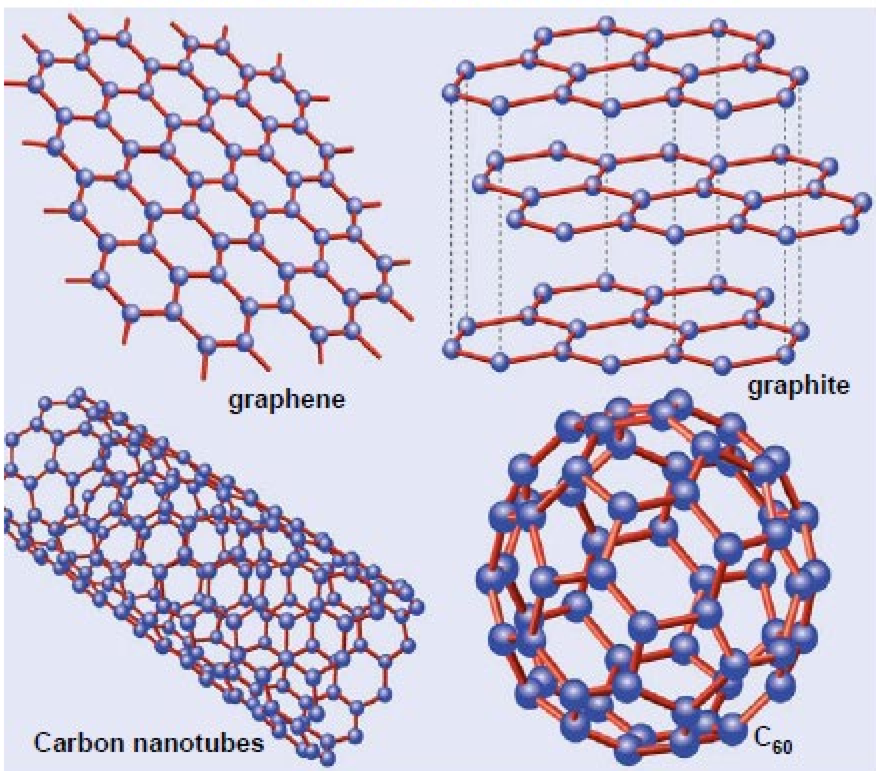




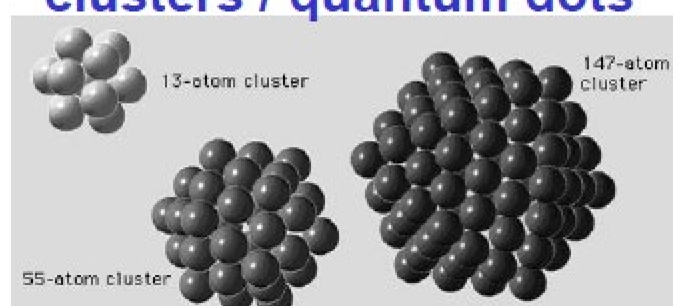




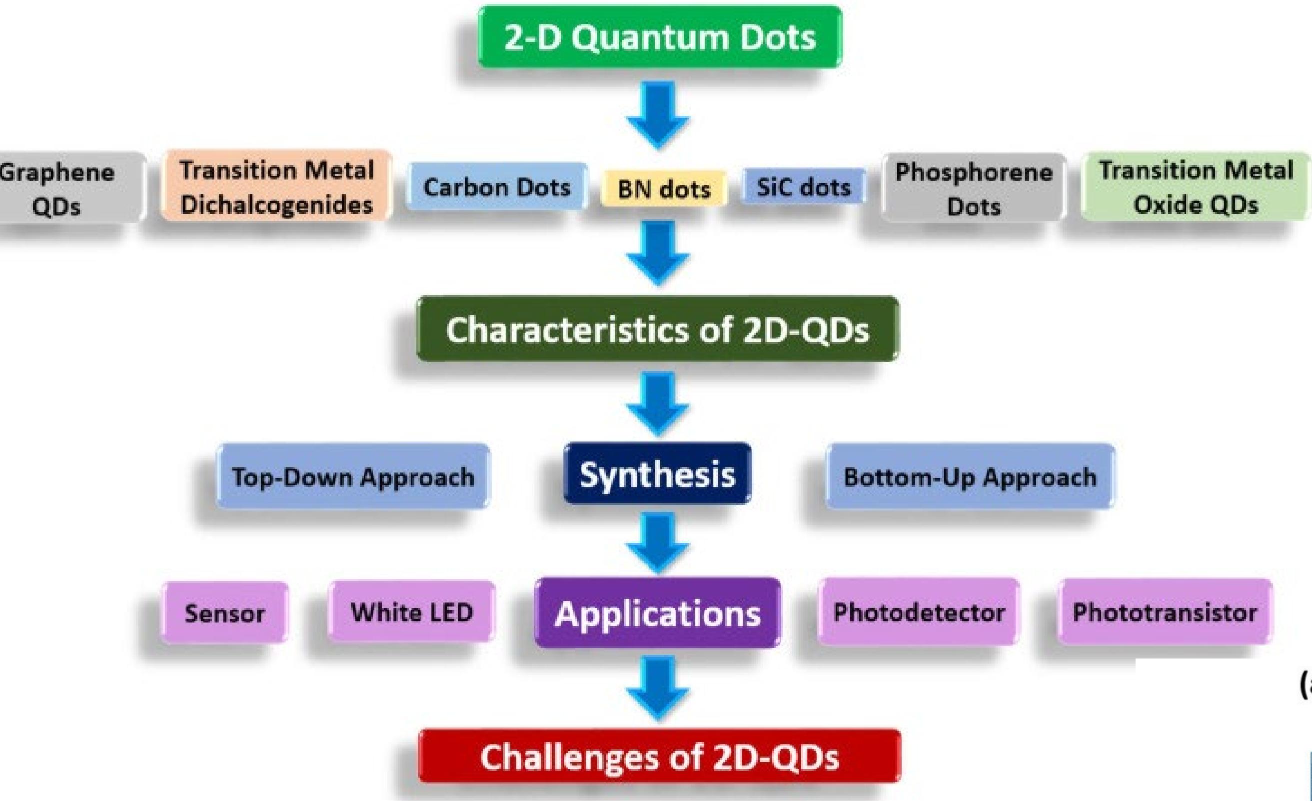
Carbon-based materials



clusters / quantum dots



2D quantum dots

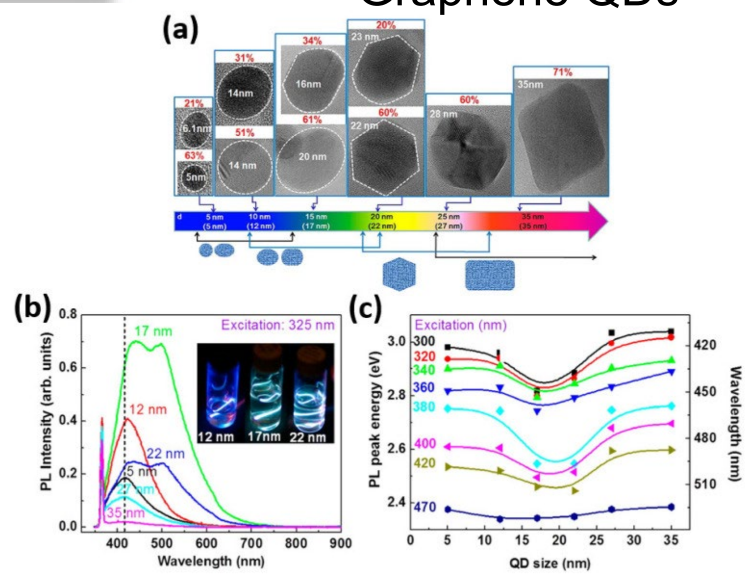


▸ Nanomaterials (Basel). 2021 Jun 11;11(6):1549. doi: 10.3390/nano11061549

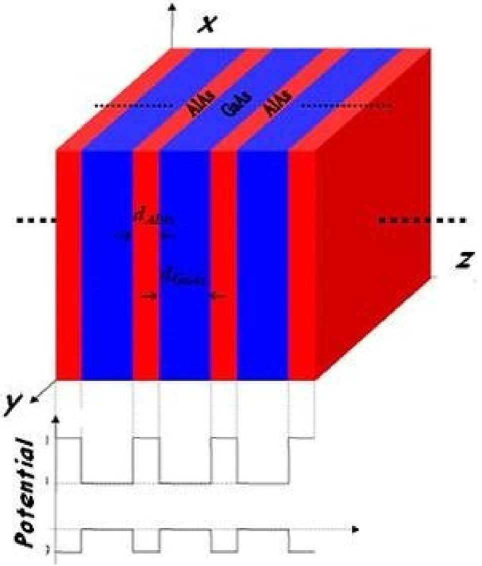
Recent Advances in Two-Dimensional Quantum Dots and Their Applications

Konthoujam James Singh [1], Tanveer Ahmed [2], Prakalp Gautam [3], Annada Sankar Sadhu [1], Der-Hsien Lien [2], Shih-Chen Chen [4,*], Yu-Lun Chueh [3,*], Hao-Chung Kuo [1,4,*]

Graphene QDs

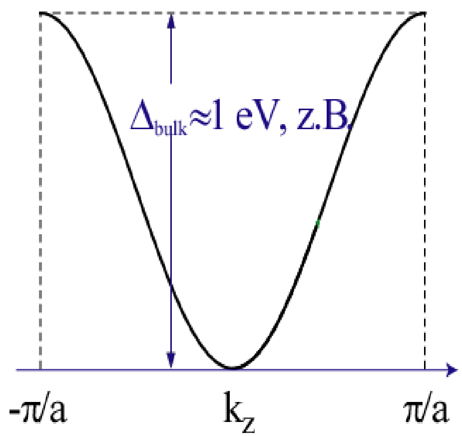


Superlattices

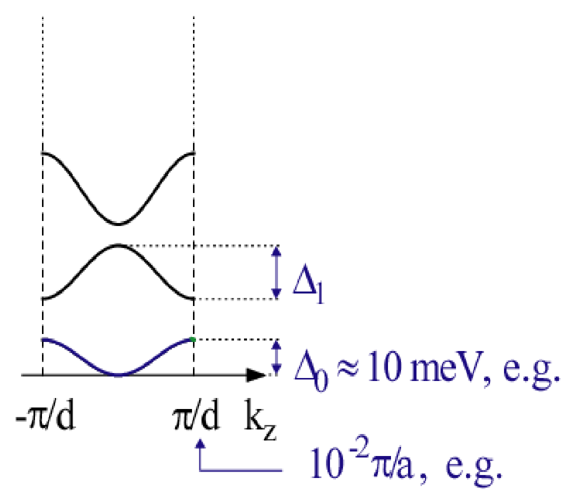


- Tunable mid-IR detectors
- Photovoltaic structures (e.g. Si with direct gap)

dispersion in bulk semiconductor
$E(k_z)$



dispersion in superlattice
$E(k_z)$



period of superlattice d >> period of semiconductor crystal a
reduced Brillouin-zone along growth direction z + lifting of degeneracy at zone boundary
=> "minibands"

tight-binding approximation: $$E(k_z) = \frac{\Delta}{2}(1 - \cos(k_z d))$$

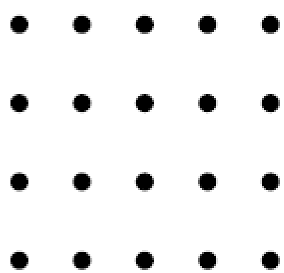
Transport in electric fields

Scattering

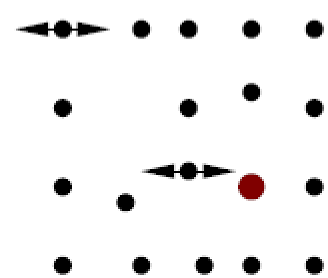
Electron propagation in real materials is NOT an uninterrupted process but is instead DISRUPTED by electron SCATTERING from a number of different sources

The origin of such scattering can be ANY source of DISORDER that destroys the perfect symmetry of the crystal structure

$\Rightarrow$ Examples of such disorder include DEFECTS and IMPURITIES in the crystal structure but scattering from other ELECTRONS as well as from the quantized LATTICE VIBRATIONS (phonons) is also possible



ELECTRONS IN A PERFECTLY PERIODIC POTENTIAL PROPAGATE WITHOUT BEING SCATTERED ... THIS WELL KNOWN RESULT IS REFERRED TO AS BLOCH'S THEOREM



IN REAL CRYSTALS HOWEVER THE PRESENCE OF INEVITABLE DISORDER DISRUPTS ELECTRON PROPAGATION THROUGH THE CRYSTAL STRUCTURE

Mean free path

An important time scale for electron transport is the **RELAXATION TIME** (τ) which is the average time over which the initial momentum of the electron is **REVERSED** through a series of scattering events in the crystal

* Using the relaxation time we may introduce the concept of the **MEAN FREE PATH** which may be defined the average **DISTANCE** electrons travel before backscattering

$$l = v_F \tau$$

* Some **IMPORTANT QUANTITIES** related to the relaxation time and mean free path include

$$Mobility\ \mu = \frac{e\tau}{m^*} = \frac{el}{\hbar k_F} \qquad 2D\ Diffusion\ Constant\ D = \frac{1}{2} v_F^2 \tau$$

$$Conductivity\ \sigma = \frac{ne^2\tau}{m^*} = ne\mu \qquad 1D\ Diffusion\ Constant\ D = v_F^2 \tau$$

* There is an elastic mean free path (ℓ_e; scattering fixed impurities and boundaries) and an inelastic mean free path (ℓ_i; scattering off phonons and other electrons).

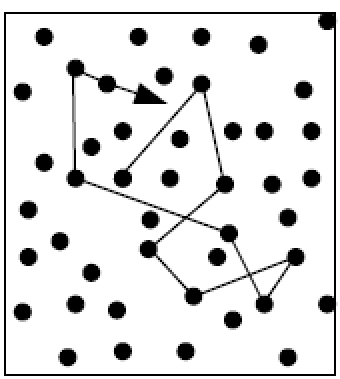
Transport regimes

Since submicron structures can now be fabricated on length scales SMALLER than the average impurity spacing in semiconductors it is possible to study electron transport in a number of different REGIMES

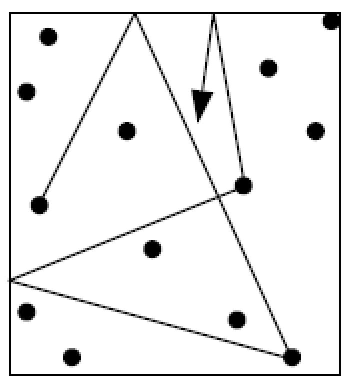
* In DIFFUSIVE conductors the mean free path is much SMALLER than the sample dimensions and DISORDER scattering dominates

* In a QUASI-BALLISTIC conductor the mean free path and device size are COMPARABLE

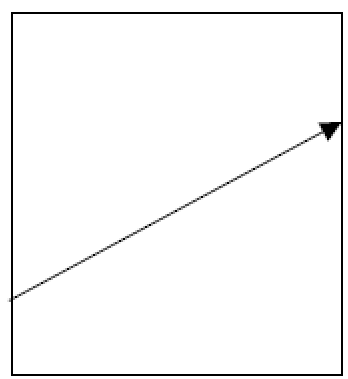
* A BALLISTIC conductor contains NO impurities and so the dominant source of electron scattering is at the device BOUNDARIES (Is the resistance zero?)



DIFFUSIVE TRANSPORT



QUASI-BALLISTIC TRANSPORT



BALLISTIC TRANSPORT

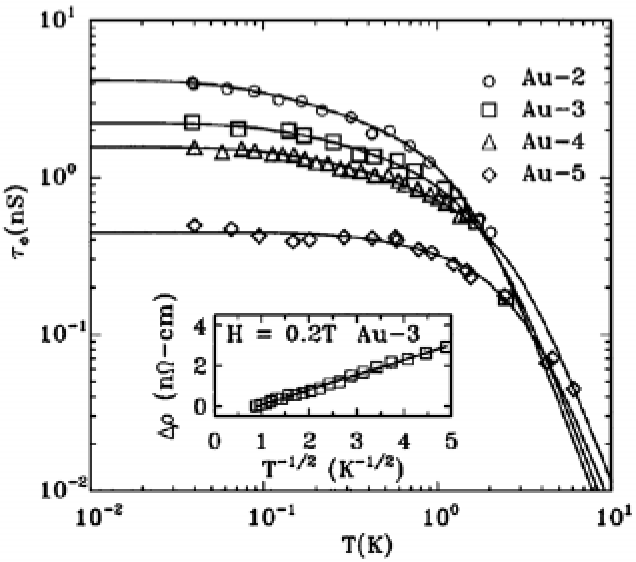
Coherent vs incoherent transport

To account for the disruption of interference effects in real materials we introduce the electron PHASE-BREAKING TIME (τ_φ) which can be thought of as the average time that elapses between dephasing events

*** The PHASE-BREAKING LENGTH (l_φ) can be defined as the average distance that electrons DIFFUSE in the material before their phase is disrupted through scattering**

$$l_\varphi = \sqrt{D\tau_\varphi} \qquad (2.4)$$

*** To observe clear interference effects it is necessary that this length is COMPARABLE to the device sizes which often requires that experiments be performed at LOW TEMPERATUES**



• THE MEASURED VARIATION OF THE PHASE-BREAKING TIME WITH TEMPERATURE IN SMALL GOLD WIRES

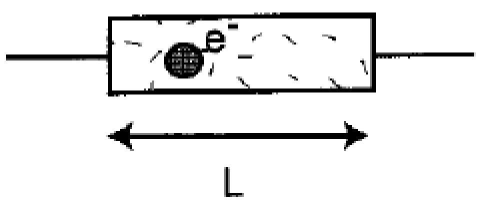
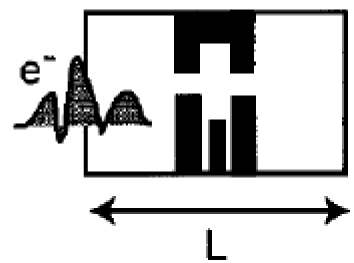
• OFTEN WE DOT DISTINGUISH BETWEEN INELASTIC MEAN FREE PATH AND THE PHASE BREAKING LENGTH. THEY ARE DIFFERENT THOUGH. WHICH ONE IS LARGER?

P. Mohanty *et al.*
Phys. Rev. Lett. 78, 3366 (1997)

Diffusive vs ballistic and classical vs quantum transport

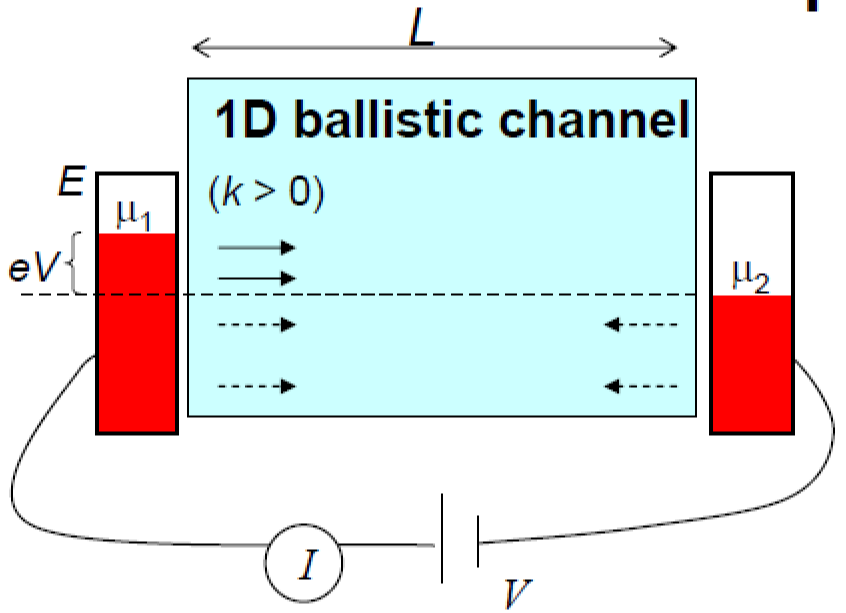
diffusive
- **classical:** $\lambda_F, \ell_i, \ell_e << L$
- **quantum:** $\lambda_F, \ell_e << L, \ell_i$

ballistic
- **classical:** $\lambda_F << L < \ell_e, \ell_i$
- **quantum:** $\lambda_F, L < \ell_e < \ell_i$

conventional device: 		mesoscopic device: 	
$L>>l_e$	diffusive	$L \lesssim l_e$	ballistic
$L>>l_\phi$	incoherent	$L \lesssim l_\phi$	phase coherent
$L>>\lambda_F$	no size quantization	$L \lesssim \lambda_F$	size quantization
$e^2/C<k_B\Theta$	no single electron charging	$e^2/C \gtrsim k_B\Theta$	single electron charging effects

		GaAs(100)	Si (100)	UNITS
Effective Mass	m	0.067	0.19	$m_e = 9.1 \times 10^{-28}$ g
Spin Degeneracy	g_s	2	2	
Valley Degeneracy	g_v	1	2	
Dielectric Constant	ε	13.1	11.9	$\varepsilon_0 = 8.9 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Density of States	$\rho(E) = g_s g_v (m/2\pi\hbar^2)$	0.28	1.59	$10^{11}\,\mathrm{cm^{-2}\,meV^{-1}}$
Electronic Sheet Density[a]	n_s	4	1-10	$10^{11}\,\mathrm{cm^{-2}}$
Fermi Wave Vector	$k_F = (4\pi n_s/g_s g_v)^{1/2}$	1.58	0.56–1.77	$10^6\,\mathrm{cm^{-1}}$
Fermi Velocity	$v_F = \hbar k_F/m$	2.7	0.34–1.1	10^7 cm/s
Fermi Energy	$E_F = (\hbar k_F)^2/2m$	14	0.63–6.3	meV
Electron Mobility[a]	μ_e	10^4–10^6	10^4	$\mathrm{cm^2/V \cdot s}$
Scattering Time	$\tau = m\mu_e/e$	0.38–38	1.1	ps
Diffusion Constant	$D = v_F^2 \tau/2$	140–14000	6.4–64	$\mathrm{cm^2/s}$
Resistivity	$\rho = (n_s e \mu_e)^{-1}$	1.6–0.016	6.3–0.63	kΩ
Fermi Wavelength	$\lambda_F = 2\pi/k_F$	40	112–35	nm
Mean Free Path	$l = v_F \tau$	10^2–10^4	37–118	nm
Phase Coherence Length[b]	$l_\phi = (D\tau_\phi)^{1/2}$	200–···	40–400	$\mathrm{nm}(T/\mathrm{K})^{-1/2}$
Thermal Length	$l_T = (\hbar D/k_B T)^{1/2}$	330–3300	70–220	$\mathrm{nm}(T/\mathrm{K})^{-1/2}$
Cyclotron Radius	$l_{cycl} = \hbar k_F/eB$	100	37–116	$\mathrm{nm}(B/\mathrm{T})^{-1}$
Magnetic Length	$l_m = (\hbar/eB)^{1/2}$	26	26	$\mathrm{nm}(B/\mathrm{T})^{-1/2}$

Conductance of 1D quantum wire



Contacts: 'Ideal reservoirs'

Chemical potential $\mu \sim E_F$
(Fermi level)

Channel: 1D, ballistic
(transport without scattering)

$$\boxed{G = I/V = \frac{2e^2}{h}}$$

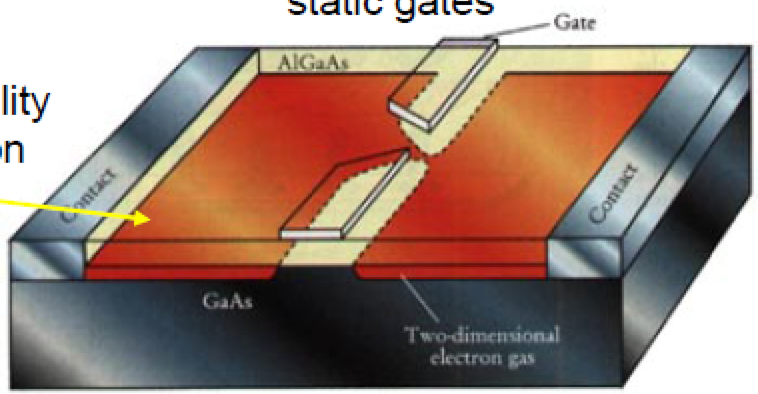
Conductance is fixed, regardless of length L, no well defined conductivity σ

Quasi-1D channel in 2D electron system

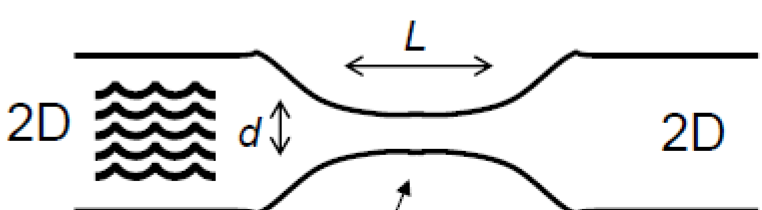
$$G(E_F) = \frac{2e^2}{h}\sum_n T_n(E_F) \approx \frac{2e^2}{h} N(E_F)\ (T = 0\mathrm{K})$$

Depletion by electro-static gates

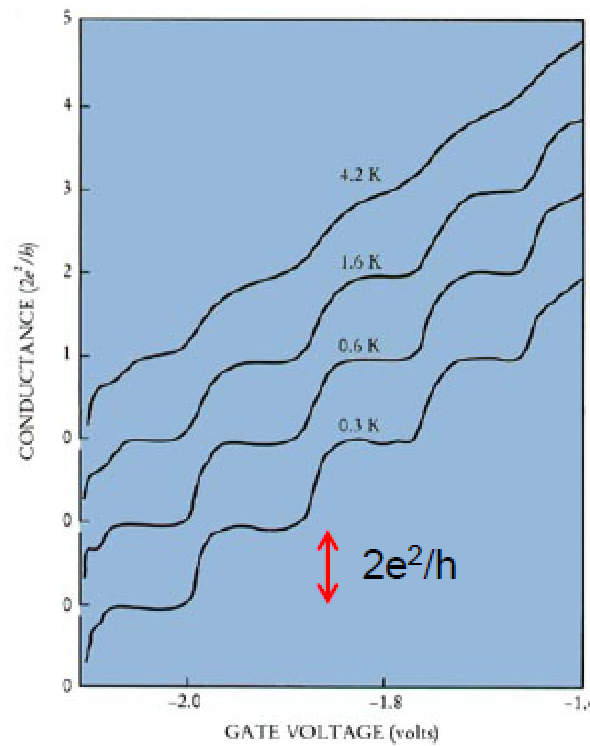
High mobility 2D electron gas ('ballistic')



'Quantum Point Contact'

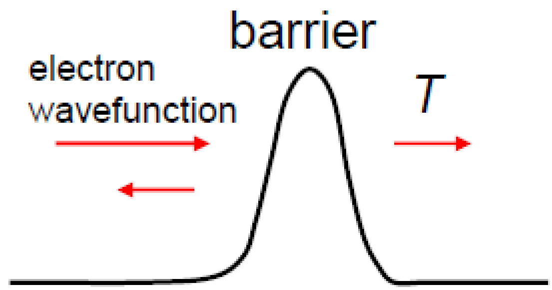


Narrow constriction; quasi-1D (width d ~ Fermi wavelength λ_F)



Limited conductance $2e^2/h$ even without scattering, regardless of length L: "contact resistance"

Conductance from transmission



Rolf Landauer (1927-1999);
- *G* controversial issue in 80'ies

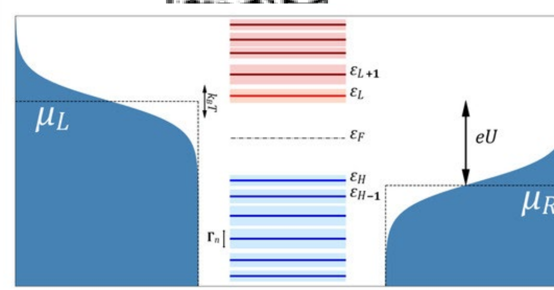
Landauer formula:

$$G = \frac{2e^2}{h} T \quad (\text{transmission probability } T)$$

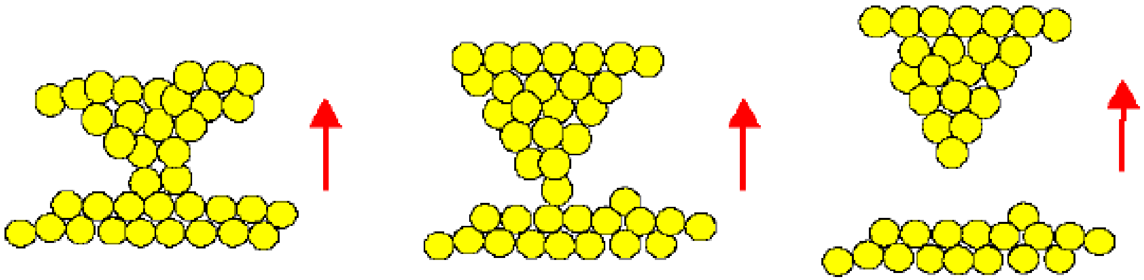
With *N* parallel 1D channels (subbands):

$$G(E_F) = \frac{2e^2}{h} \sum T_n(E_F) \;\; (T = 0)$$

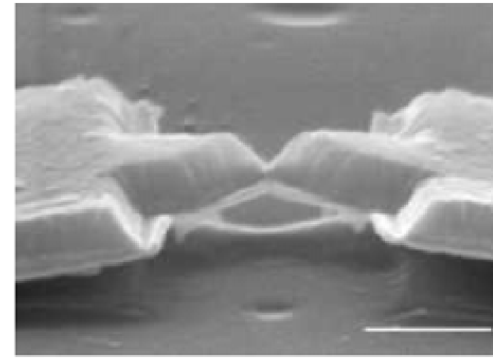
$$G(T) \approx \frac{2e^2}{h} \int_{-\infty}^{\infty} T(E) \left(- \frac{\partial f(E)}{\partial E} \right) dE \vee$$

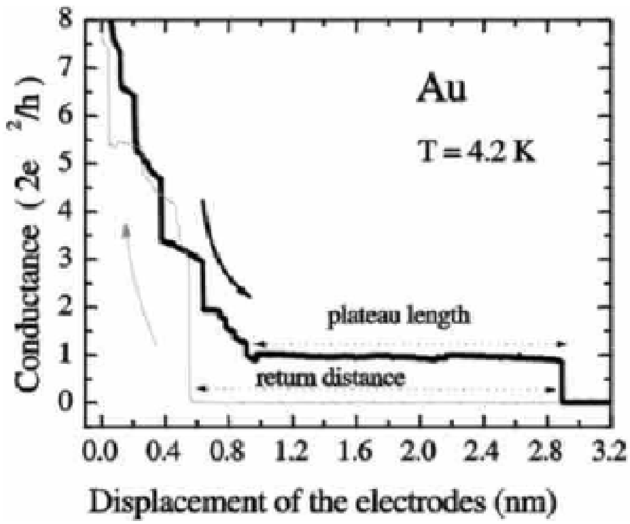


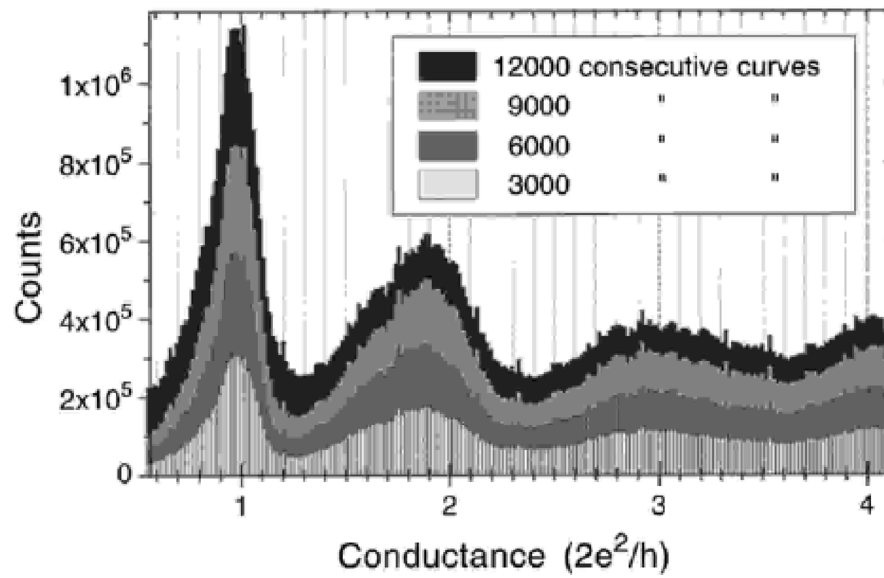
Break junctions



elongation monoatomic contact rupture







Overview break junctions: N. Agrait, A.L. Yeyati, J.M. van Ruitenbeek, Physics Reports 377, 81 (2003)

Nanowire formation in macroscopic metallic contacts: quantum mechanical conductance tapping a table top

J.L. Costa-Krämer [a], N. García [a], P. García-Mochales [a], P.A. Serena [b]

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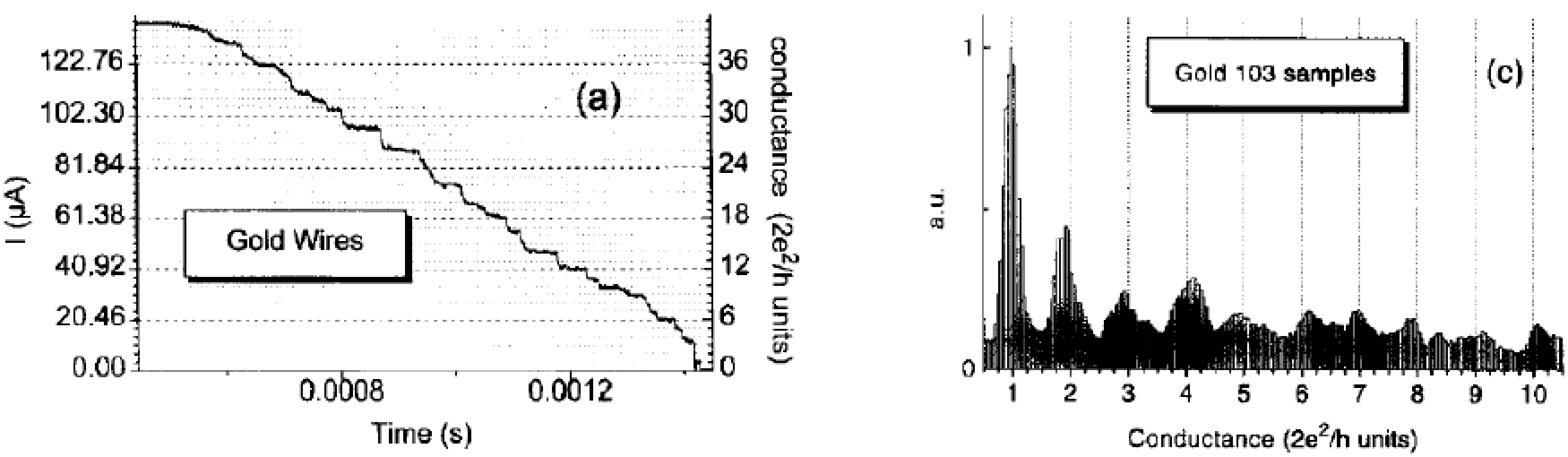
Surface Science 342 (1995) L1144–L1149

Abstract

In this letter we show that quantum mechanical conductance is observed in nanowires formed by placing two wires of macroscopic dimensions in contact, making them vibrate so they get in and out of contact, and measuring the conductance response of such a system with an oscilloscope. We do this by tapping the table top on which the loose contact formed by the macroscopic wires is placed. The formation of these nanowires and the associated quantized conductance is a universal phenomenon that occurs when any two metals get in and out of contact independently of the metal sizes. This should have strong technological implications in studying contact formation, friction, tribology, forming and breaking bonds, mechanics, etc., at the nanoscopic level. Results and simple specifications needed for the formation of nanowires are presented for Au, Cu, Pt and metallic glass wires.



single-electron tunneling (SET)

- classical dots (SET islands): level spacing is NOT important; only the charging energy (=classical effect, many electrons on the island)

- quantum dots: level spacing (quantum confinement) AND charging energy important (few electrons on the dot)

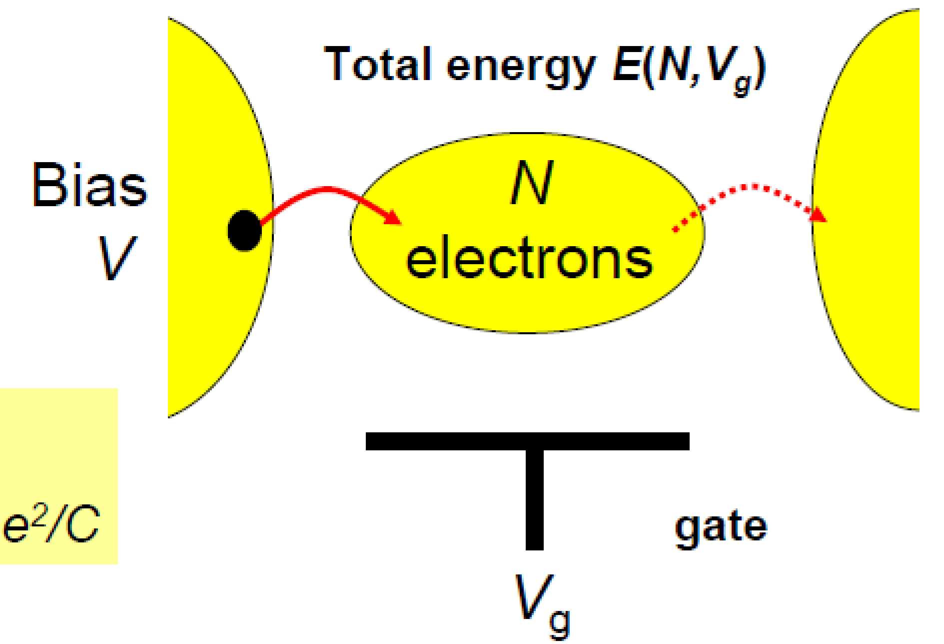
Coulomb blockade

Electrons can tunnel only at V_g for which

$E(N+1, V_g) = E(N, V_g) \pm kT$

Cost for adding one electron:

charging energy: $E_C \sim Q^2/C \sim e^2/C$

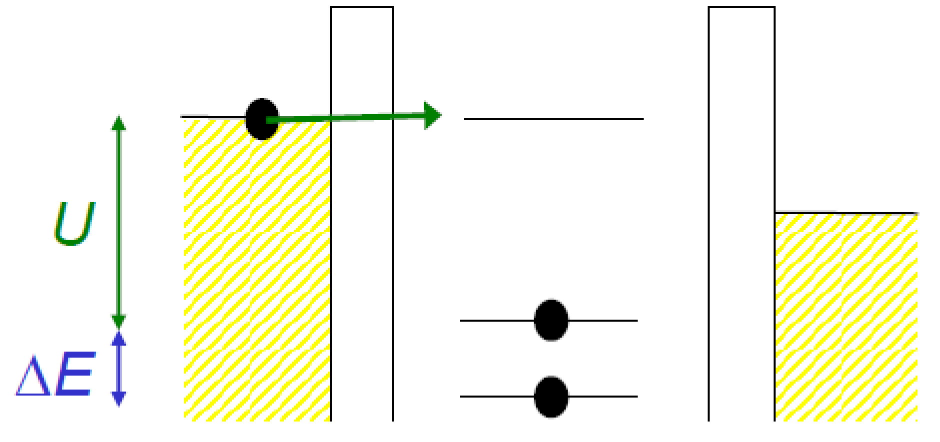


Quantum dots
(artificial atoms)

Two energy parameters:

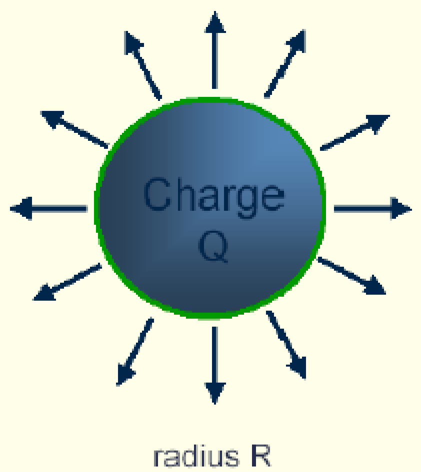
U – 'charging energy' e^2/C
(e-e interaction strength)

ΔE – single-particle level spacing



charging energy: $E_C=e^2/2C$

- What is the capacitance of an isolated piece of metal (for example a sphere)?



Electric field:

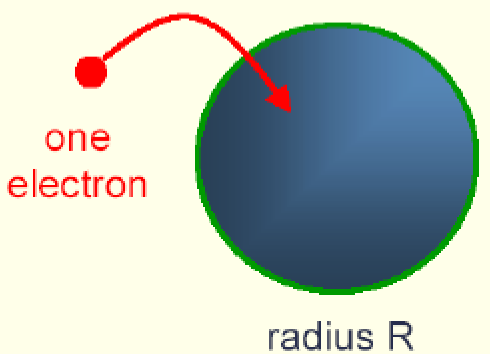
$$\vec{E}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \quad (r > R)$$

Voltage:

$$V(R) = -\int_R^\infty \vec{E}(\vec{r})\cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0 R}$$

$$C = Q/V = 4\pi\varepsilon_0 R$$

- What is the energy needed to charge the sphere with one electron (1/2QV with Q = e)?

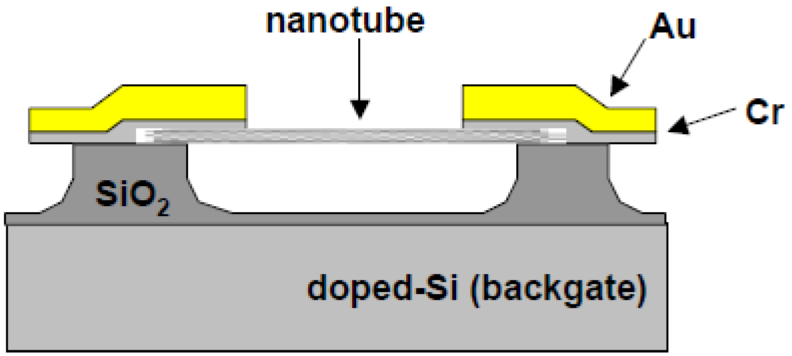


R	C	E/k_B
10 μm	1.1×10^{-15} F	0.84 K (^{3}He)
1 μm	1.1×10^{-16} F	8.4 K (LHe)
0.1 μm	1.1×10^{-17} F	84 K (LN_2)
0.01 μm	1.1×10^{-18} F	840 K (spa)

$C = 4\pi\varepsilon_0 R$

capacitances

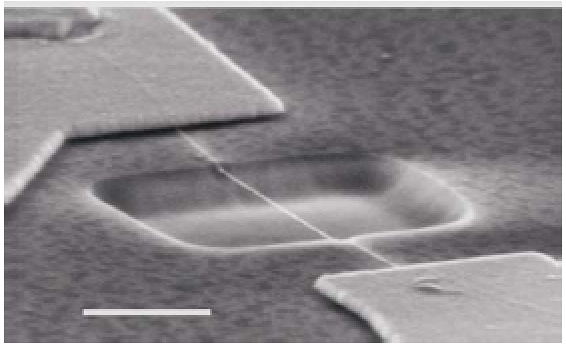
isolated sphere (dot): $C_{sphere} = \varepsilon_0\varepsilon_r\, 2\pi d$



isolated disk: $C_{disk} = \varepsilon_0\varepsilon_r\, 4d$

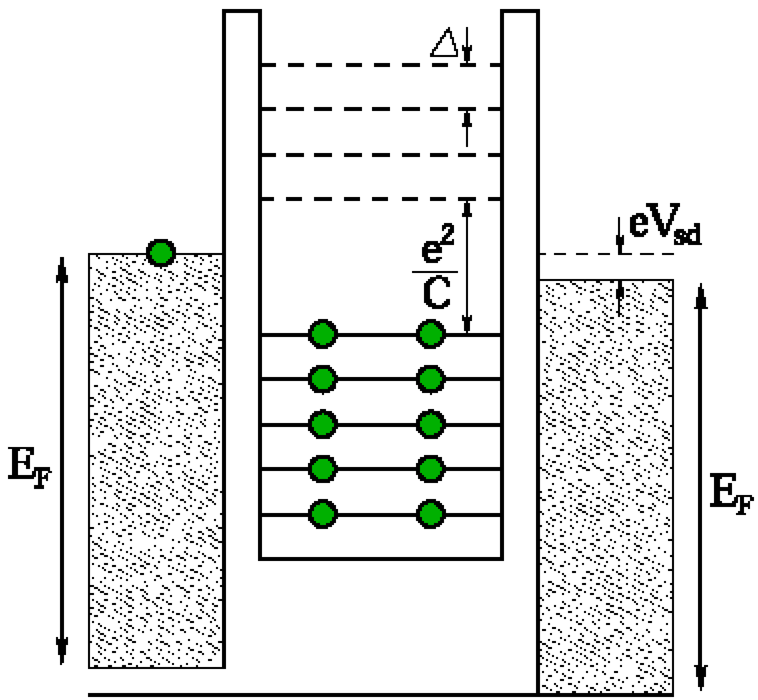
parallel plate: $C_{parallel\ plate} = \varepsilon_0\varepsilon_r\, A/d$

nanotube with diameter, r, above a ground plane at distance h: $C_{NT} = \varepsilon_0\varepsilon_r\, 2\,\pi\, L\, /\, \ln(2h/r)$

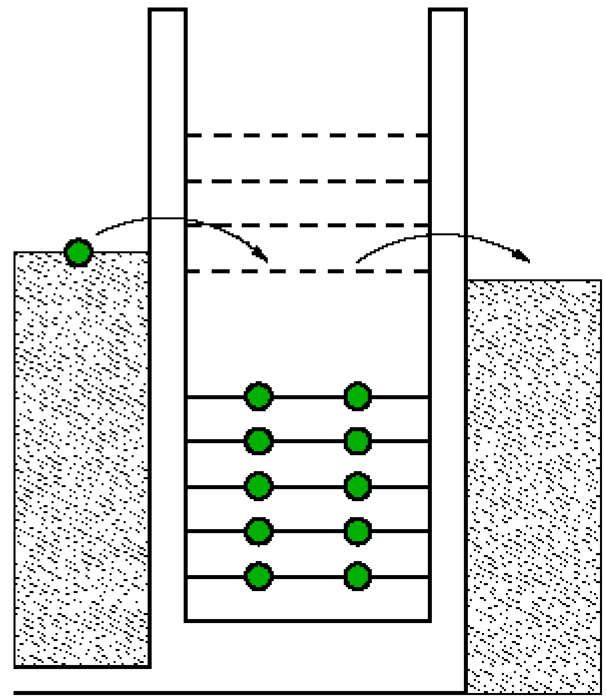


quick estimate: capacitance per unit length: $C' = \varepsilon_0\varepsilon_r = \varepsilon_r\, 10\ aF/\mu m$

Coulomb blockade



No current



Current

Few-Electron Quantum Dots in Nanowires

Mikael T. Björk,[†,§] Claes Thelander,[†,§] Adam E. Hansen,[†] Linus E. Jensen,[†]
Magnus W. Larsson,[‡] L. Reine Wallenberg,[‡] and Lars Samuelson[*,†]

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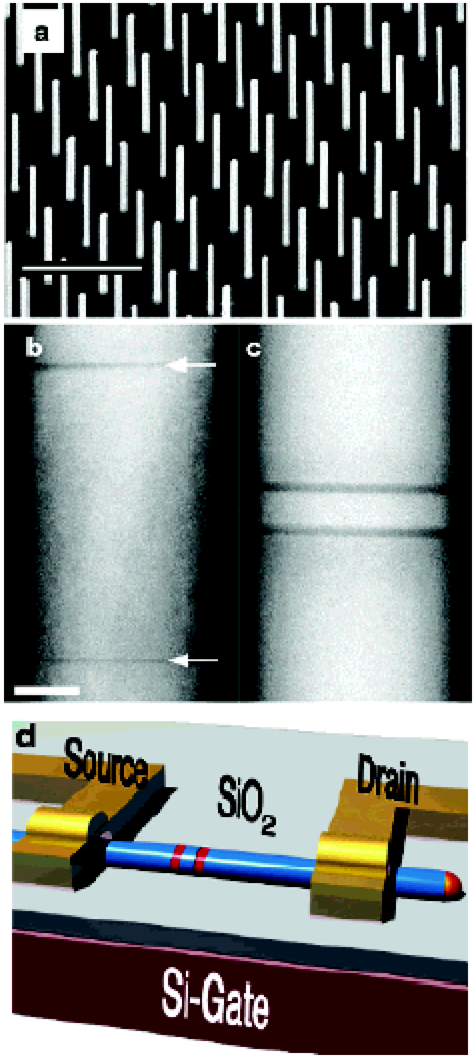
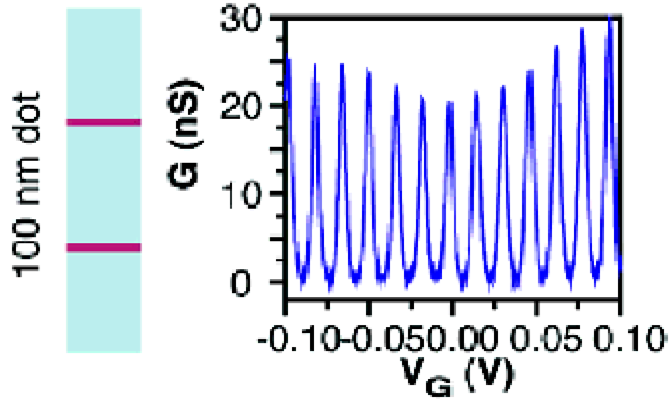
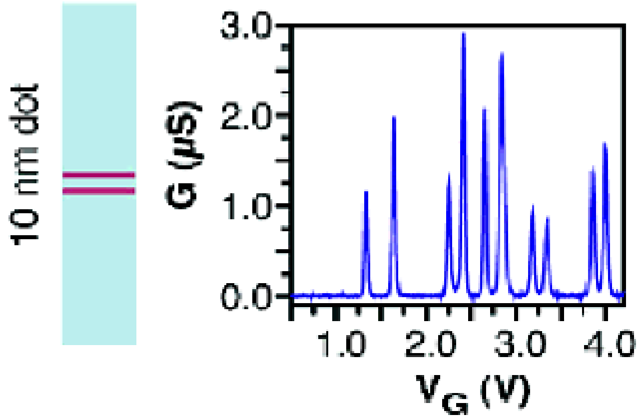


classical dot: regular spaced Coulomb peaks



quantum dot: irregular spaced Coulomb peaks



VOLUME 67, NUMBER 12 PHYSICAL REVIEW LETTERS 16 SEPTEMBER 1991

Quantized Current in a Quantum-Dot Turnstile Using Oscillating Tunnel Barriers

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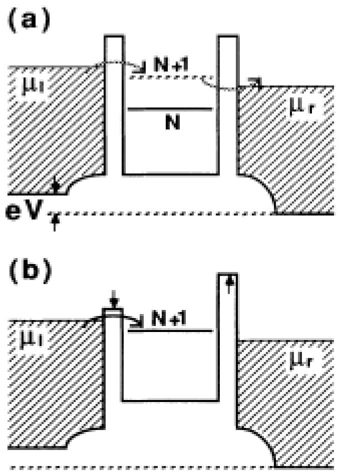
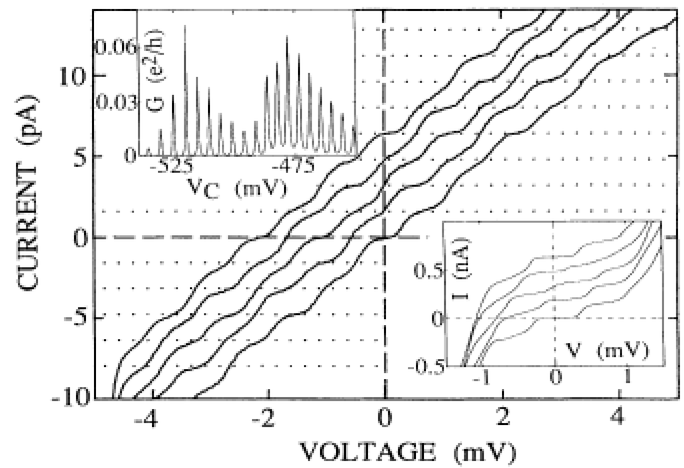
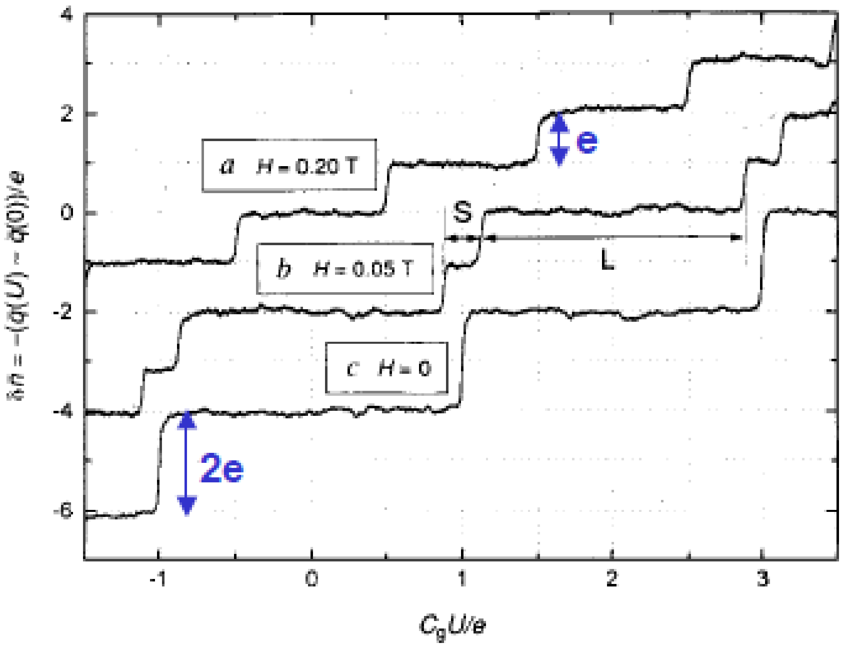
(Received 20 May 1991)





Homework !

measured charge quantization in a normal and superconducting single-electron box



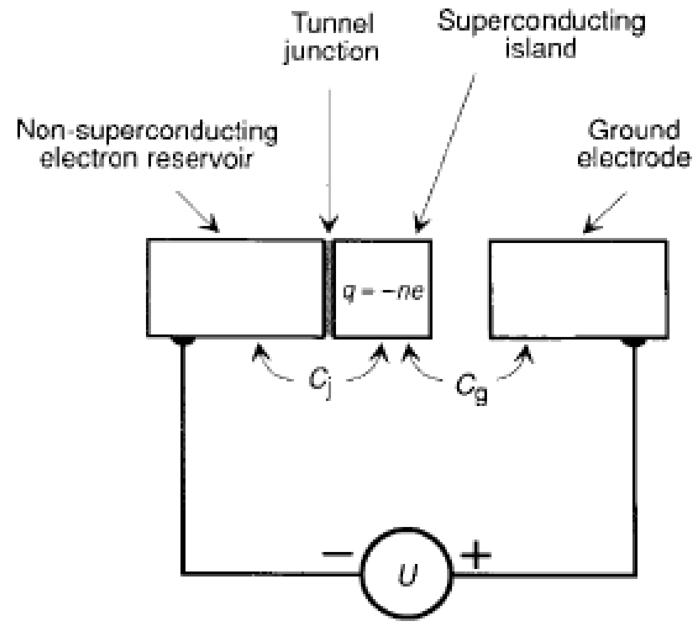


FIG. 1 Schematic diagram of the experiment. The superconducting island is a 30 × 110 × 2,260 nm Al strip containing ~10^9 atoms. Its dimensions are such that the electrostatic energy of one extra electron is much larger than the energy k_BT of thermal fluctuations at temperature $T \sim 30$ mK. The island can exchange electrons with a Cu (3 wt% Al) thin-film electrode (which acts as an electron reservoir) through a tunnel junction[17]. The total charge q of the island varies under the influence of the externally controlled voltage source U connected between the electron reservoir and a ground electrode. The variation with U of the time average q of the island charge is measured by a Coulomb blockade electrometer (not shown) which is weakly capacitively coupled to the island. The nanofabrication and low-noise measurement techniques involved in this type of experiment have been described in refs 5 and 18.

Two-electron quantization of the charge on a superconductor

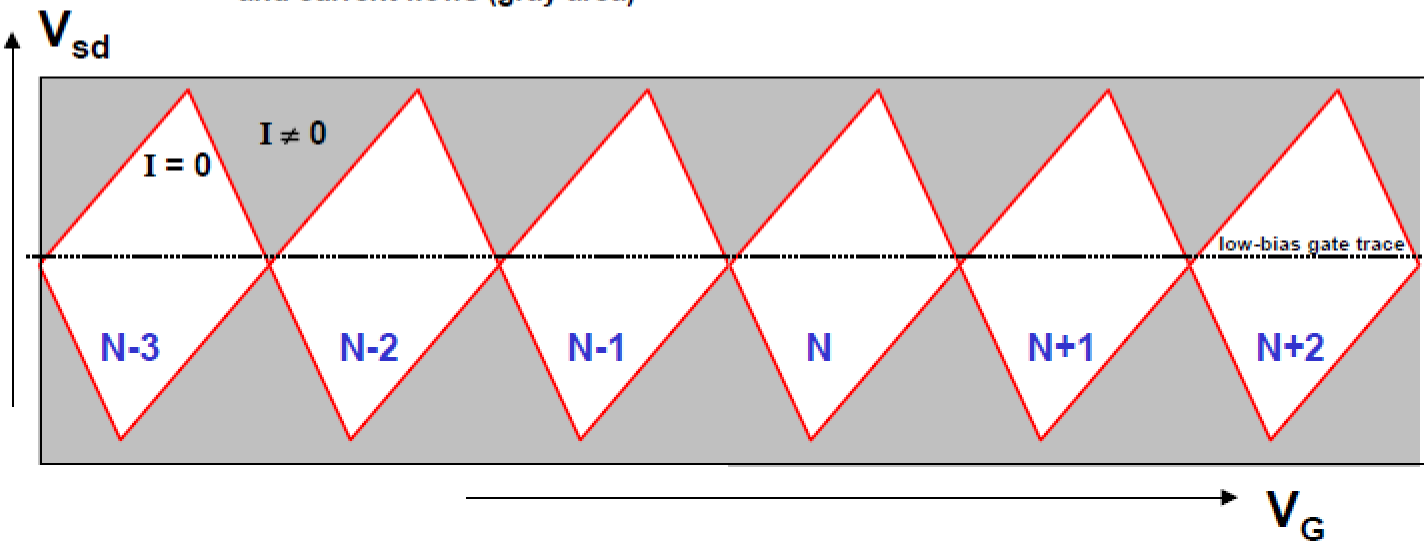
P. Lafarge, P. Joyez, D. Esteve, C. Urbina & M. H. Devoret

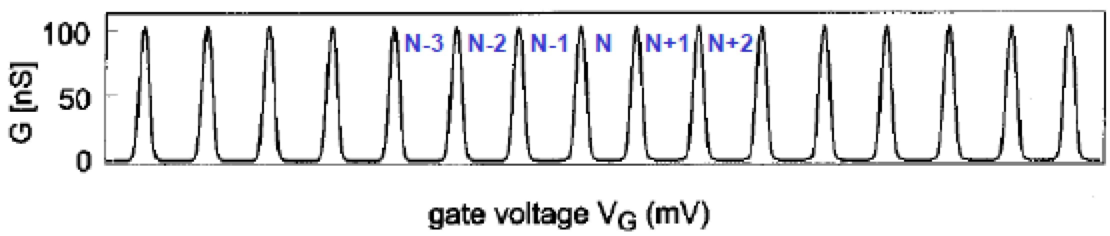
NATURE · VOL 365 · 30 SEPTEMBER 1993

gate traces and stability diagram

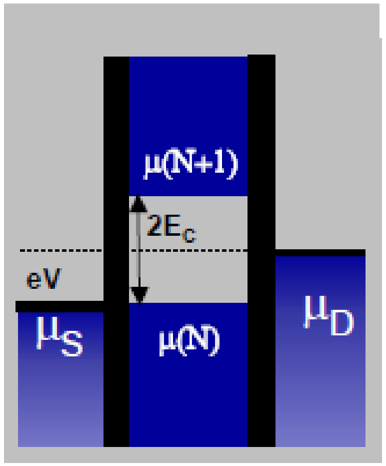
inside the Coulomb diamonds: the number of electrons on the island is fixed and no current flows

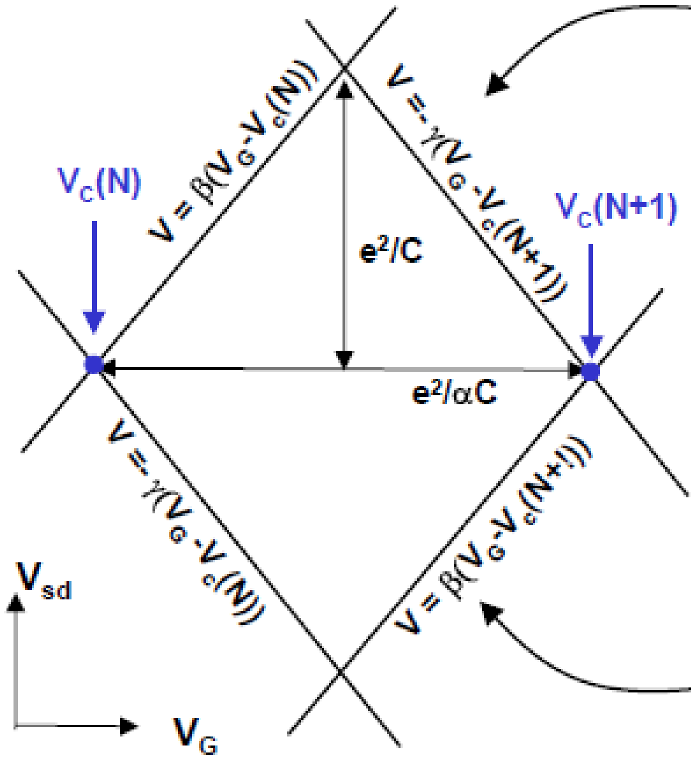
outside the Coulomb islands: the number of electrons fluctuates and current flows (gray area)

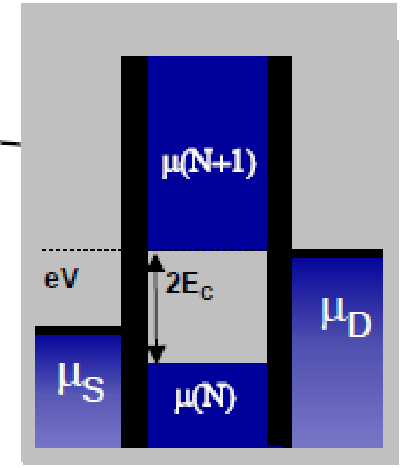


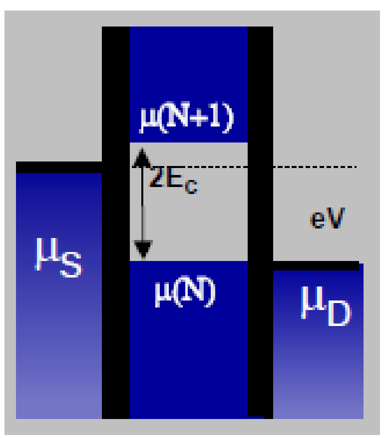


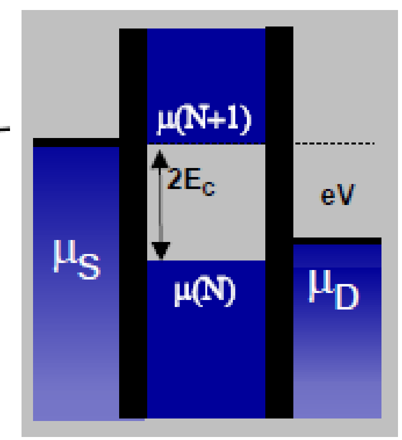
Coulomb diamonds











The lines define a region in which there is no current. This region is called the Coulomb diamond. At zero bias, current flows at the degeneracy points indicated in blue below.

Coulomb diamonds: the equations

From $\mu_S = \mu(N)$ we find $V = \beta(V_G - V_C)$ with $\beta = C_G/(C_G + C_D)$

From $\mu_D = \mu(N) = 0$ we find $V = -\gamma(V_G - V_C)$ with $\gamma = C_G / C_S$

$V_C = (N - 1/2)e/C$, i.e. the voltage corresponding to the chemical potential on the dot in the absence of an external potential.

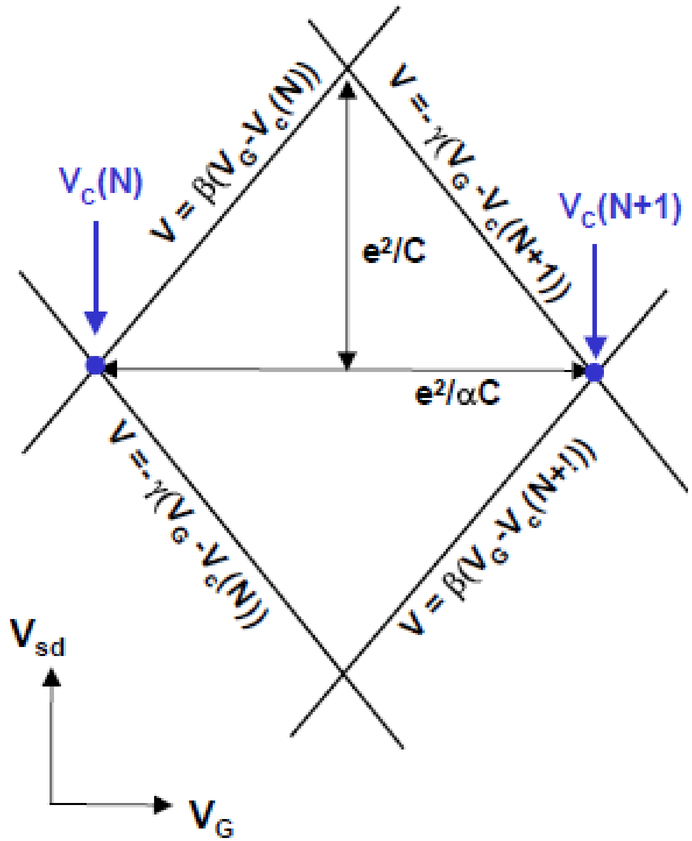
The energy required to put an extra electron on the island (having already N-electrons is called the addition energy:

$$E_{add} = \mu(N+1) - \mu(N) = \frac{e^2}{C} = 2E_C$$

In a measurement the addition energy can be read off from the height of the Coulomb diamonds or from the distance between adjacent crossing point ($V_C(N+1) - V_C(N) = e/C_G = 2E_C/\alpha$).

The latter term contains a factor α, which is the gate coupling parameter: the potential on the island varies linearly with the gate voltage, $\Delta V_I = \alpha \Delta V_G$ where

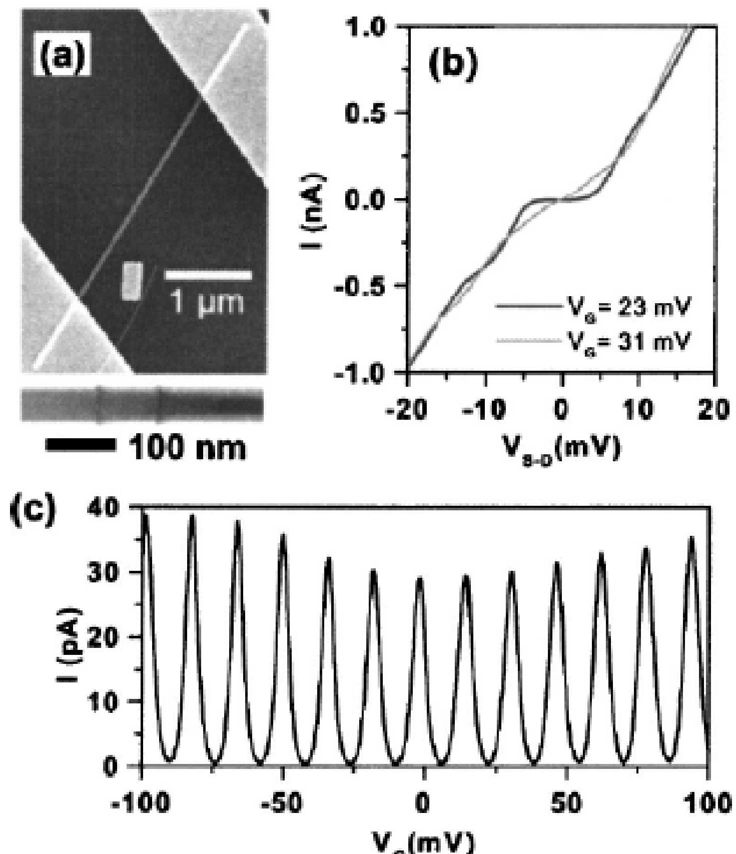
$$\frac{C}{C_G} = \frac{1}{\alpha} = \frac{1}{\beta} + \frac{1}{\gamma}$$

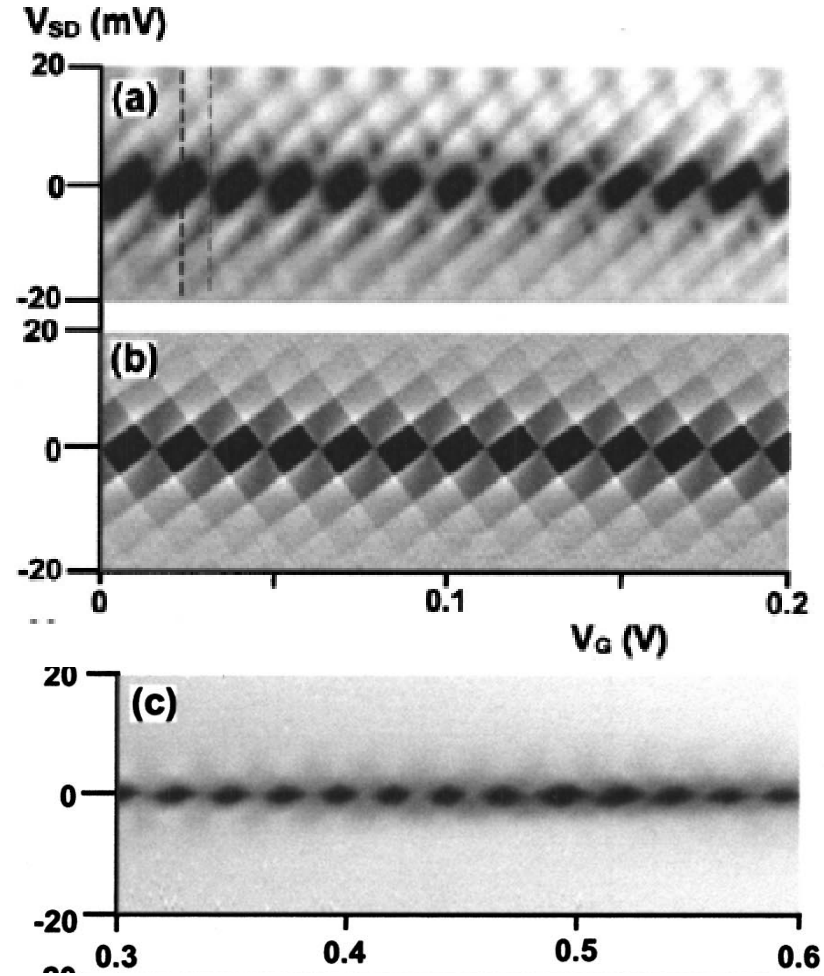


APPLIED PHYSICS LETTERS VOLUME 83, NUMBER 10 8 SEPTEMBER 2003

Single-electron transistors in heterostructure nanowires

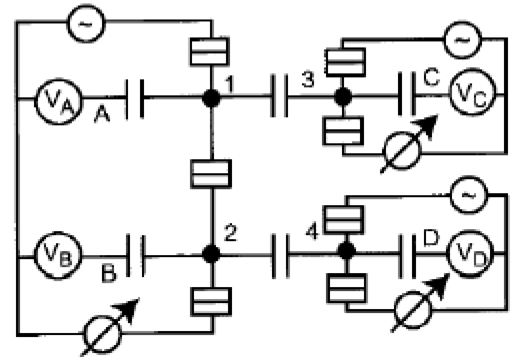
C. Thelander,[a)] T. Mårtensson, M. T. Björk, B. J. Ohlsson, M. W. Larsson,[b)]
L. R. Wallenberg,[b)] and L. Samuelson
Solid State Physics/Nanometer Consortium, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden





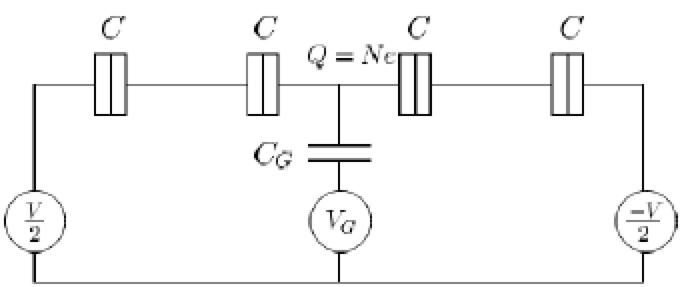
SET applications

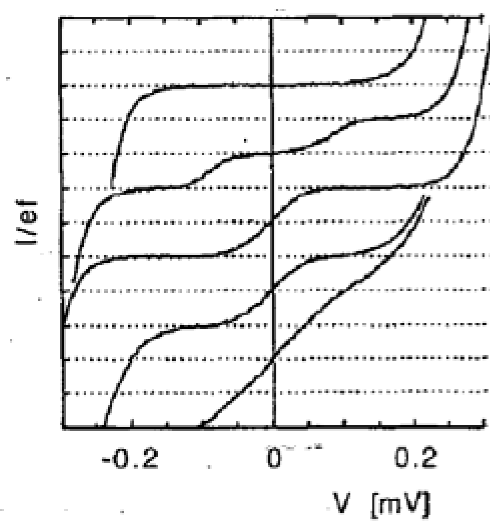
- sensitive charge measurements (10^{-5} e/√Hz)

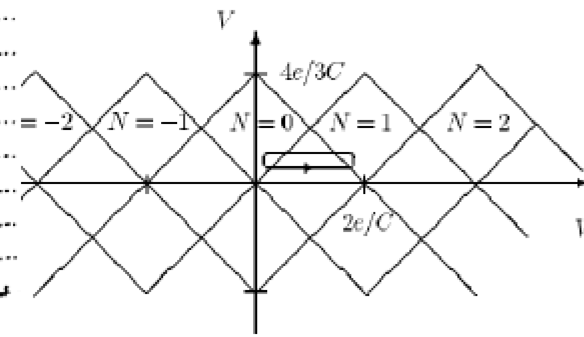


Experiment: SETs 3 and 4 work as electrometers to measure charges at the islands 1 and 2

- current standard (turnstile)







- Single-electron logic, memory

Single-electron logic and memory devices

ALEXANDER N. KOROTKOV†‡

INT. J. ELECTRONICS, 1999, VOL. 86, NO. 5, 511–547

issues:
random offset charges
room-temperature operation

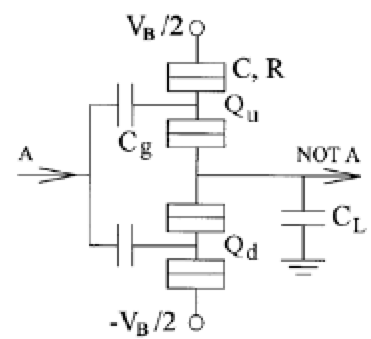
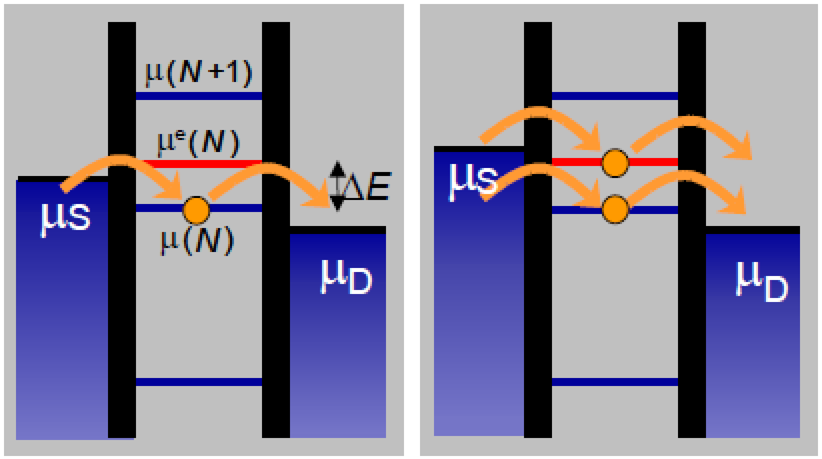
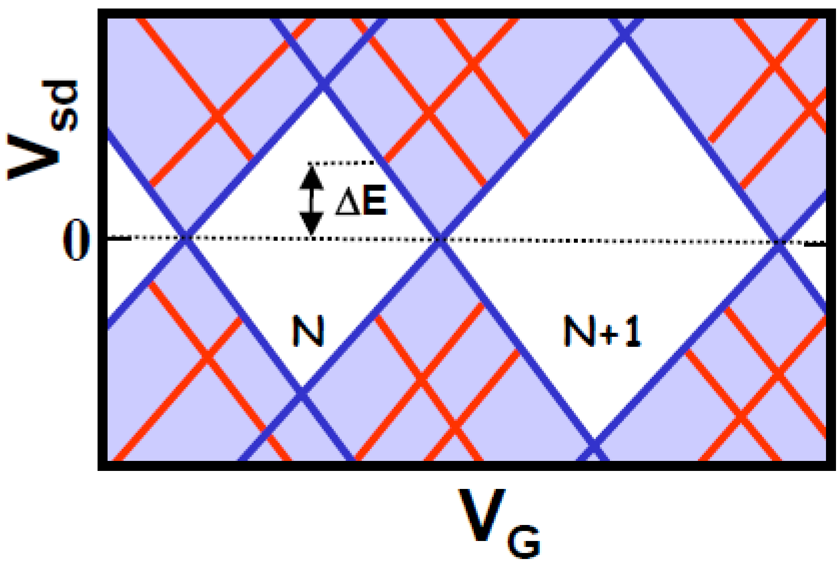


Figure 5. The complementary inverter made of two SET-transistors.

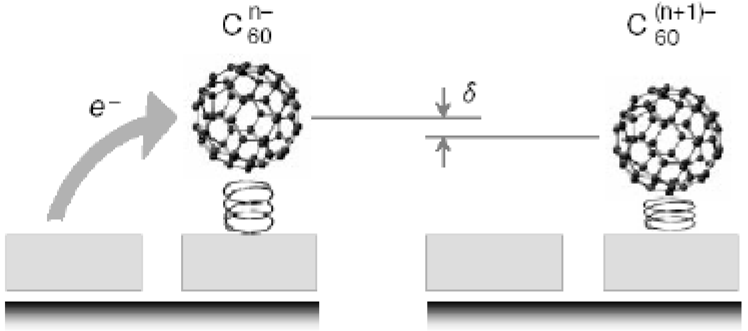
level spectroscopy: excited states





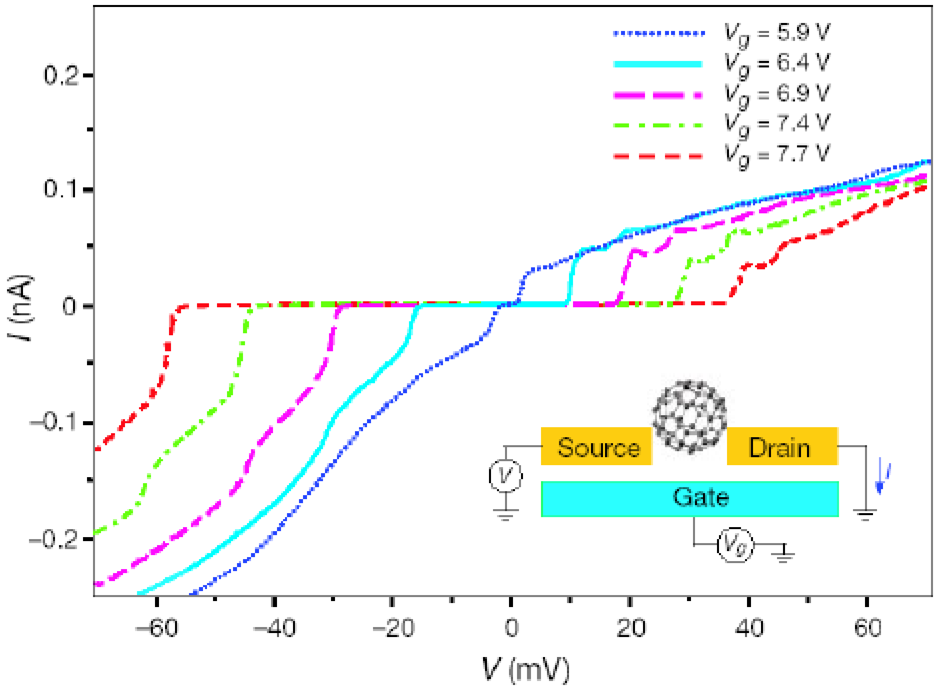
- when an excited level enters the transport window, an additional transport channel opens up leading to a step-wise increase of the current. In the differential resistance (which is often plotted in the stability diagram), these steps appear as lines running parallel to the diamond edges (red lines)
- the energy of the excited state can directly be read off from the diagram as indicated in the figure
- excitations can probe electronic spectrum, spin or vibrational states

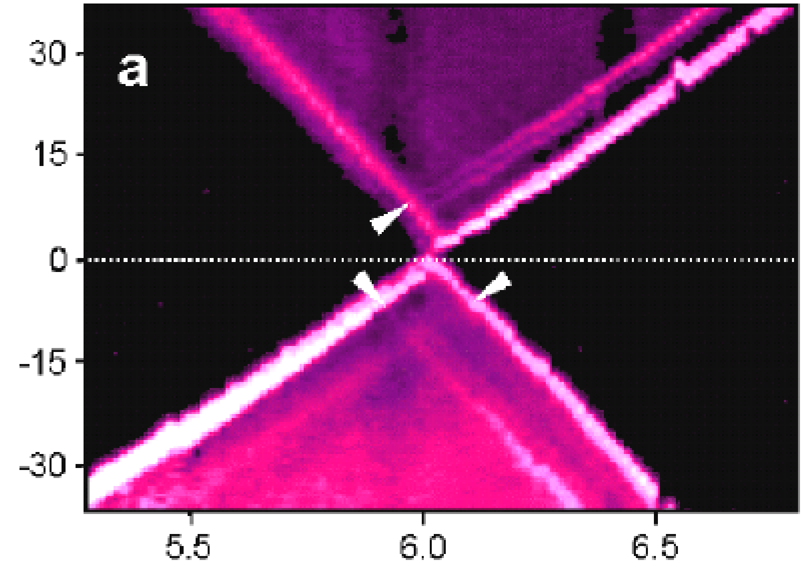
vibration assisted tunnelling in a C_{60} transistor



single electron tunneling events excite and probe the mechanical motion of the C60 bucky ball

vibrational mode adds another transport channel: step in current-voltage characteristic





H. Park et al. Nature ***407****, 57 (2000)*