Lecture #2

Low-dimensional semiconductor systems

Quantum wells, wires, & dots

Suppose electron is confined within a box with dimensions L_x, L_y, L_z $\Rightarrow$ introduce characteristic length λ (Lecture #1)

(i) $\lambda \ll L_{x,y,z} \Rightarrow$ 3D bulk semiconductor

(ii) $\lambda > L_x$, $L_x \ll L_{y,z} \Rightarrow$ 2D semiconductor, $\perp x$ $\Downarrow$ quantum well

(iii) $\lambda > L_{x,y}$, $L_{x,y} \ll L_z \Rightarrow$ 1D semiconductor, or quantum wire along z

(iv) $\lambda \gg L_{x,y,z} \Rightarrow$ 0D semiconductor, or quantum dot

What are density-of-states in each of these cases?

Recall: $N_{3D}(E) \sim \sqrt{E}$
(Lecture #1)

②

2D

QW

$$N_{2D}(E)\,dE = \frac{1}{A}\cdot 2 \cdot \frac{2\pi k\,dk}{\frac{(2\pi)^2}{A}} = \frac{k}{\pi}\,dk$$

per unit area   spin

$$N_{2D}(E) = \frac{k}{\pi}\frac{dk}{dE} = \frac{k}{\pi}\frac{m_e^*}{\hbar^2 k} = \frac{m^*}{\pi\hbar^2}$$

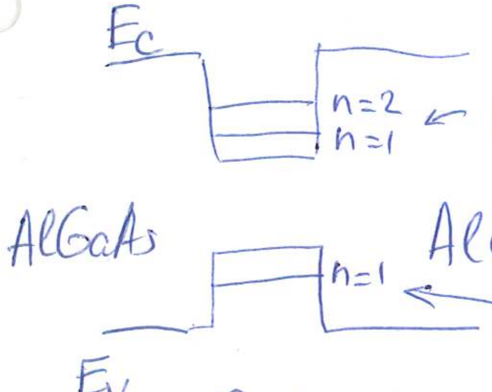
$$\frac{dE}{dk} = \frac{\hbar^2 k}{m_e^*}$$

DOS function is a constant!

For a 2D system,

$$E(k_x, k_y, n) = \frac{\hbar^2}{2m_e^*}(k_x^2 + k_y^2) + \underbrace{E_n}_{=\frac{\hbar^2\pi^2}{2m_e^* a^2}n^2}$$

particle in the box

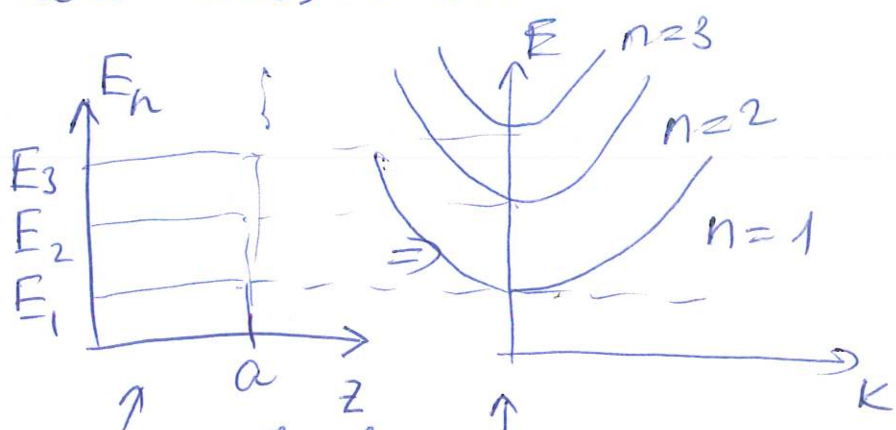


electron levels in QW

AlGaAs (~2 eV)

hole levels in QW

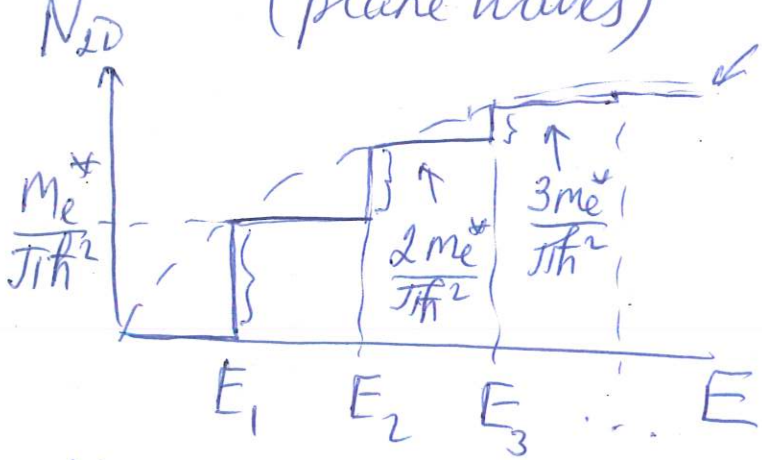
GaAs (1.4 eV)



energy levels in QW

energy sub-bands

$$\Psi = \underbrace{\Psi_x \Psi_y}_{\substack{\text{free}\\ \text{electron}\\ \text{(plane waves)}}} \Psi_z \leftarrow \sqrt{\frac{2}{a}} \sin\frac{\pi n z}{a}$$

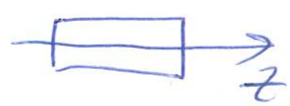


(all steps the same height!)

$$E_F = \frac{\pi\hbar^2}{m_e^*} n_e$$

Note: QW doesn't have to be square $\Rightarrow$ can be parabolic, triangular, etc $\Rightarrow$ different E_n!

$\boxed{1D}$ Quantum wires



$$\Psi = e^{ik_z z} u(x,y)$$

$$E_{n_1,n_2}(k_z) = \underbrace{E_{n_1,n_2}}_{\frac{\hbar^2\pi^2}{2m_e^*}\left(\frac{n_1^2}{a_x^2} + \frac{n_2^2}{a_y^2}\right)} + \frac{\hbar^2 k_z^2}{2m_e^*}$$

$\leftarrow$ if a square potential, $n_{1,2} = 1, 2, 3, \ldots$

$$N_{1D}(E)\,dE = \underset{\substack{\uparrow\\ \text{per length}}}{\frac{1}{L}} \cdot \underset{\substack{\uparrow\\ \text{spin}}}{2} \cdot \frac{dk}{\frac{2\pi}{L}} = \frac{1}{\pi}\,dk$$

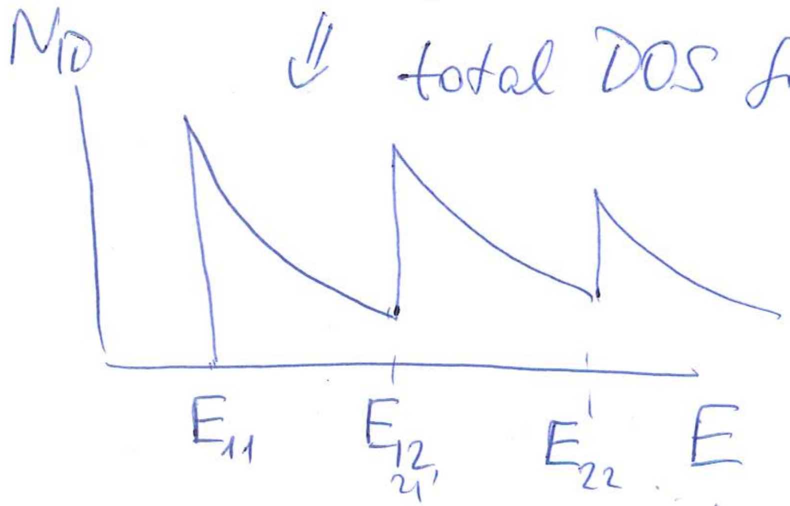
$$N_{1D}(E) = \frac{1}{\pi}\frac{dk}{dE} = \frac{1}{\pi}\frac{m_e^*}{\hbar^2 k} = \frac{1}{\pi}\frac{m_e^*}{\hbar^2}\frac{\hbar}{\sqrt{2m_e^* E}} =$$

$$= \sqrt{\frac{m_e^*}{2}}\,\frac{1}{\pi\hbar}\,\frac{1}{\sqrt{E}} \qquad (\times 2)$$

$\uparrow$ k + or −

$\uparrow$ diverges for $E=0$

$\Downarrow$ total DOS for 1D;



$$N_{1D}(E) = \sum_{n_1,n_2} \frac{1}{\pi\hbar}\sqrt{\frac{2m_e^*}{E-E_{n_1,n_2}}}$$

$$E_F = \frac{\hbar^2}{2m^*}\left(\frac{\pi n_e}{2}\right)^2$$

Group velocity $v_g = \frac{1}{\hbar}\frac{dE}{dk} = \frac{\hbar k}{m_e^*} = \frac{1}{\pi\hbar}\frac{1}{N_{1D}(E)}$

$\Downarrow$ will use this!

Consider current

$$I = e\int_{E_{F_1}}^{E_{F_2}} N_{1D}(E)\, v_g(E)\,dE = \frac{e}{\pi\hbar}\underbrace{(E_{F_2}-E_{F_1})}_{=eV} = \frac{2e^2}{h}V$$

$\boxed{0D}$ Quantum dots ← a.k.a. artificial atoms

nanocrystals with all three spatial dimensions in the nm range; $\sim 10^4 - 10^6$ atoms; $\lambda_B \sim$ Q.D. size

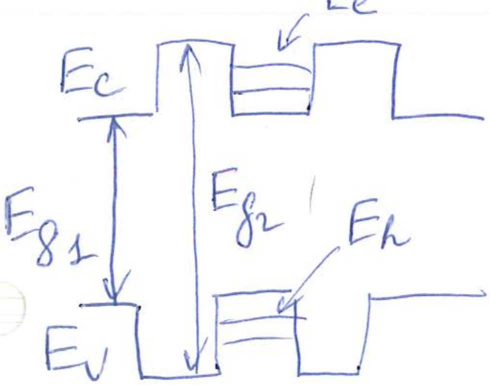
$$E_{n_{1,2,3}} = \frac{\hbar^2 \pi^2}{2 m_e^*} \left(\frac{n_1^2}{a_x^2} + \frac{n_2^2}{a_y^2} + \frac{n_3^2}{a_z^2} \right), \quad n_{1,2,3} = 1, 2, 3, \ldots$$

← energy is completely quantized

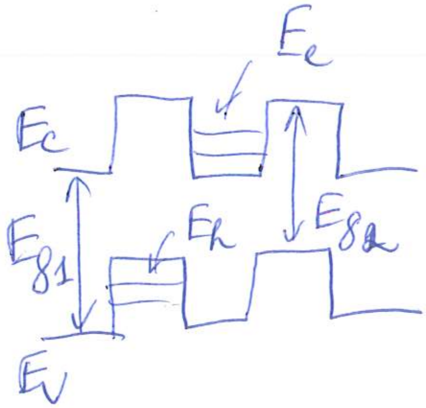
Analog of ionization energy – "charging energy" of the QD – energy of addition or removal of electron from the QD

Other nanostructures

– Multiple quantum wells (MQW) ⇒ neighboring QW are separated by >10nm; non-interacting



Type I

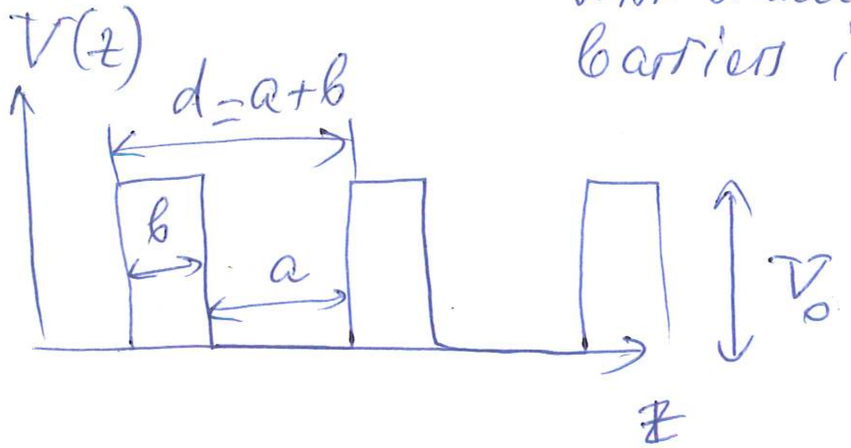


Type II

Typically:
~ 50 wells
20 nm periodicity

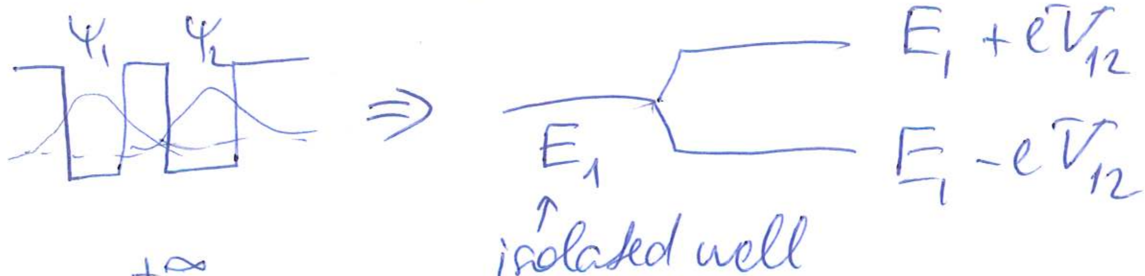
Use: IR detectors, modulators, other optoelectronics

Superlattices ← periodic set of MQWs with small thickness-energy barriers in between



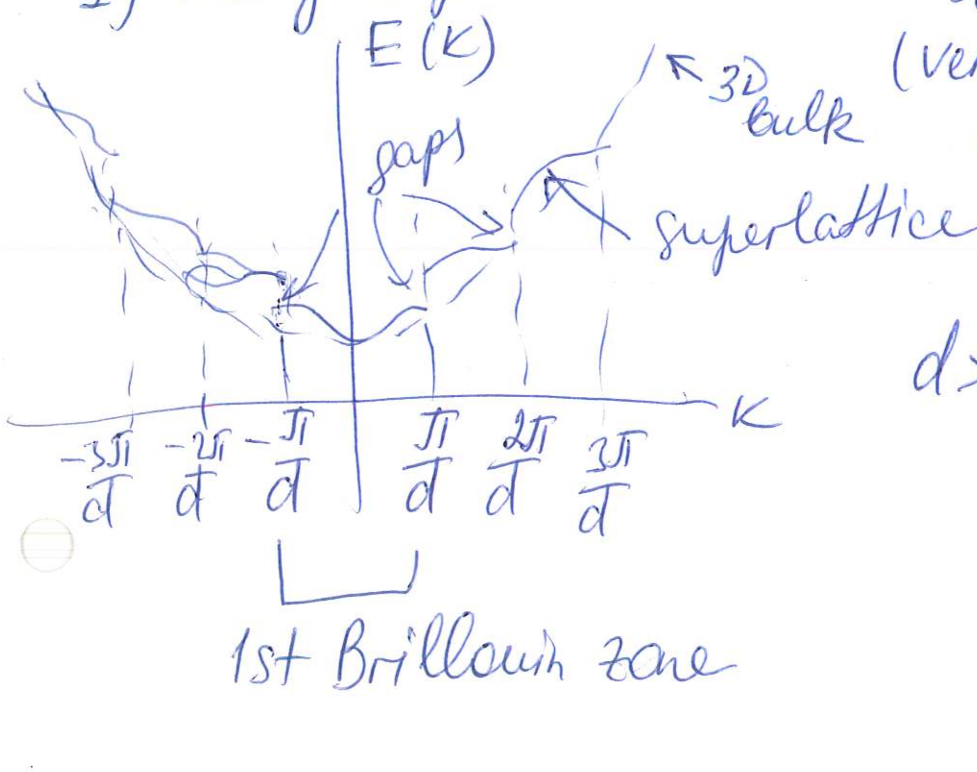
⇒ Kronig-Penney model

Consider 2 coupled wells



$$V_{12} = \int_{-\infty}^{+\infty} \psi_1^* V(z)\, \psi_2 \, dz$$

If many coupled wells ⇒ energy <u>bands</u> (very narrow compared to crystals)



d >> lattice constant in crystals

Electric field transport in nanostructures (7)

Quantized conductance

$$I = e \underbrace{N_{1D}(E)}_{\text{DOS}} \underbrace{v_g}_{\text{velocity of electron}} \overset{eV \leftarrow \text{voltage}}{E} =$$

↑ current across the wire

$$\text{DOS} = \frac{2}{2\pi\hbar v_g}$$

↑ since electrons move in only one direction

$$= \boxed{\frac{2e^2}{h}} V = GV$$

↑ fundamental conductance

$G_0 = \frac{e^2}{h}$ ← quantum unit of conductance

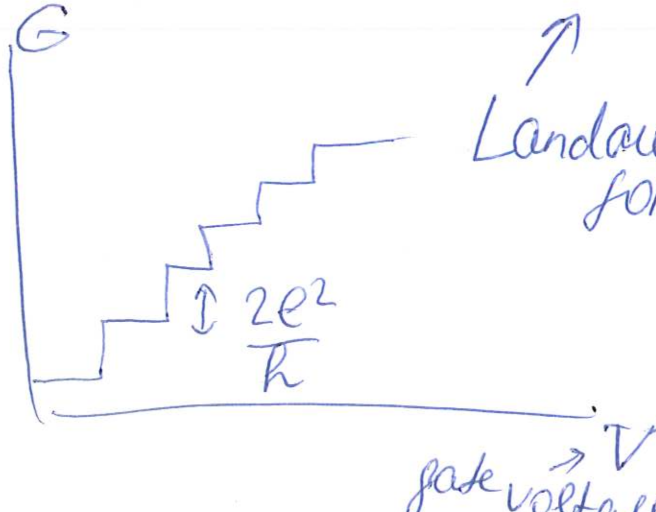
$R_0 = \frac{h}{e^2} = 25.812807\ k\Omega$ ← quantum resistance

Total conductance $G = \frac{2e^2}{h} \sum_{n,m}^{N} \underbrace{|t_{nm}|^2}$

N ← # channels contributing to conductance

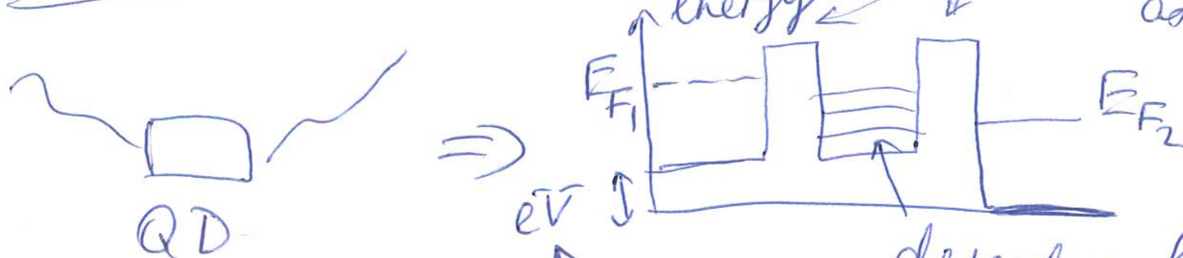
↗ Landauer formula

↑ transmission probability



← quantum conductance at 0.6K for the point contact in AlGaAs-GaAs heterojunction

Coulomb blockade

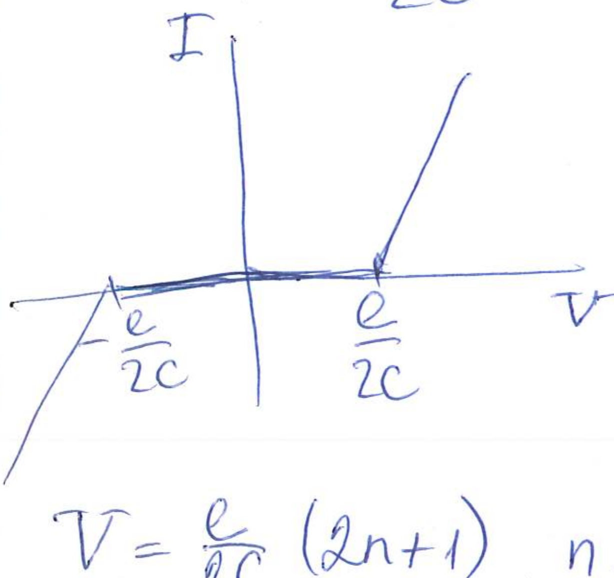


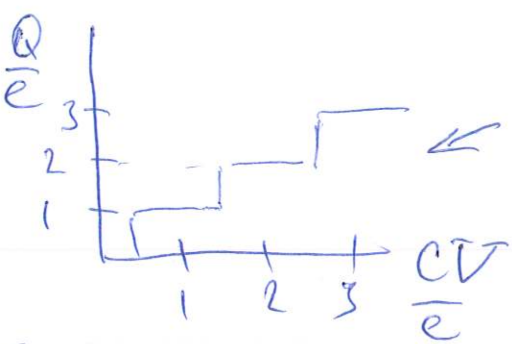
To add electron to the dot $\Rightarrow$ provide energy

increase voltage by $\frac{e}{2C}$ $\Leftarrow$ $\frac{Q^2}{2C} = \frac{e^2}{2C}$

dot capacitance

If $|V| < \frac{e}{2C} \Rightarrow$ current can't flow through the dot $\Rightarrow$ Coulomb blockade





increase V $\Downarrow$ add electrons

$V = \frac{e}{2C}(2n+1)$, $n = 0, 1, 2, \ldots$

<u>Note</u>: for a measurable effect, need $C \ll \frac{e^2}{k_B T} \Rightarrow$ either $T < 1K$ or $C < 10^{-16} F$

<u>Also</u>: $\Delta E \Delta t = \frac{e^2}{C} R_T C > h \Rightarrow R_T \gg \frac{h}{e^2} = 25{,}8 k\Omega$

tunneling resistance (R_T); quantum resistance ($\frac{h}{e^2}$)