

Homework #2

due Wednesday, November 3, 2025

1. (5 pts) Consider p-type Si with a carrier density of $1.4 \times 10^{16} \text{ cm}^{-3}$ and hole effective mass of $0.37 m_0$, where m_0 is the free electron mass. What is the plasma frequency of such a material? What do you expect from reflection and transmission of this semiconductor at optical frequencies?
2. (5 pts) Calculate the binding energy and radius of the $n = 1$ exciton in ZnS, which has $m_e^* = 0.28 m_0$, $m_h^* = 0.5 m_0$ and $\epsilon = 7.9$. What is the highest temperature at which this exciton can be observed?
3. (10 pts) We would like to optically study exciton-exciton collisions in GaN at 300 K, which has $m_e^* = 0.2 m_0$, $m_h^* = 1.2 m_0$ and $\epsilon = 10$.
 - (a) What is the lowest excitation photon energy needed to excite the $n = 1$ exciton? (E_g of GaN at 300 K is 3.4 eV).
 - (b) Assuming that each excitation photon will produce an exciton, estimate the lowest energy of the laser pulse at a wavelength calculated in (a) needed to observe exciton-exciton collisions in a 1 mm x 1 mm sample with a thickness of 100 nm.
4. (10 pts) Work through the Nature 2003 paper by Barnes et al.
 - (a) Derive Eqs.(1) and (2)
 - (b) Discuss Eq.(4) and relationship between data in Fig.4 and Eq.(1). Does Eq.(4) support the peak position in Fig. 4?
5. (10 pts) Investigate the dispersion relation for the surface plasmon polariton (SPP) (i.e. frequency ω as a function of wave number k_x). How does SPP behave at low k_x and at high k_x ? What is determined by the dielectric permittivities ϵ_1 and ϵ_2 ? Discuss the difference in SPP propagation if you consider:
 - (a) Au/air vs Ag/air interface
 - (b) Au/air vs Au/glass interface
6. (10 pts) Work through the PRL 2000 paper by Pendry. Discuss differences in transmission through a slab with "normal" refraction and with negative refraction.
7. Reading assignment: Science **302**, 419 (2003) (plasmons in complex nanostructures) and Science **305**, 847 (2004) (plasmon in arrays).

Additional reading regarding physics of metal nanoparticles and nanostructures:

- 1) L. Novotny and B. Hecht, "Principles of nano-optics" (Cambridge Univ. Press, Cambridge, 2006)
- 2) "Surface Plasmon nanophotonics", M. L. Brongersma and P.G. Kik (Eds.), (Springer, 2007)
- 3) H. Raether, "Surface plasmons on smooth and rough surfaces and on gratings" (Springer, 1998)
- 4) P. Prasad, "Nanophotonics" (Wiley-Interscience, 2004)
- 5) "Near-field optics and surface plasmon polaritons", S. Kawata (Ed.), (Springer, 2001)