**Phys 551** **Winter 2026**

**Worksheet #8**

(Friday, January 23, 2026)

**Name**

**Questions (5 pts):**

A quantum harmonic oscillator is in a state described by the following wavefunction: $|\Psi(t)\rangle = 1/\sqrt{2}\,(|0\rangle + e^{-i2\omega t}|2\rangle)$, where $|n\rangle$ are eigenstates of the Hamiltonian.

(a) What is the expectation value of the position operator X in this state? $\sqrt{\frac{\hbar}{2m\omega}}\,(a^{+}+a)$

$$\langle\Psi(t)|X|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left(\langle 0| + e^{i2\omega t}\langle 2|\right)\hat{X}\,\frac{1}{\sqrt{2}}\left(|0\rangle + e^{-i2\omega t}|2\rangle\right)$$

$$= \frac{1}{2}\sqrt{\frac{\hbar}{2m\omega}}\left(\langle 0| + e^{i2\omega t}\langle 2|\right)(a^{+}+a)\left(|0\rangle + e^{-i2\omega t}|2\rangle\right)$$

$|1\rangle$ $=0$ $\Rightarrow |1\rangle$

$$= \frac{1}{2}\sqrt{\frac{\hbar}{2m\omega}}\left(\langle 0| + e^{i2\omega t}\langle 2|\right)\left(0 + |1\rangle + e^{-i2\omega t}\sqrt{2}|1\rangle + e^{-i2\omega t}\sqrt{3}|3\rangle\right) = 0$$

$\langle 0|1\rangle = 0 \quad \langle 0|3\rangle = 0$

$\langle 2|1\rangle = 0 \quad \langle 2|3\rangle = 0$

(b) Generalize your observation to an arbitrary state

$|\Psi(t)\rangle =$ norm. factor * $(c_n e^{-in\omega t}|n\rangle + c_m e^{-im\omega t}|m\rangle)$. Under what conditions is $\langle X\rangle(t) \neq 0$?

$\langle X(t)\rangle \neq 0$ only if $m = n \pm 1$