Phys 451 **Winter 2026**

Worksheet #7

(Wednesday, January 21, 2026)

Name

Questions (5 pts):

A quantum harmonic oscillator is in a state described by the following wavefunction: $\Psi(x) = 1/\sqrt{2}\,(\varphi_0(x) + \varphi_1(x))$, where $\varphi_n(x)$ are eigenfunctions of the Hamiltonian.

(a) If energy is measured, what are the outcomes and their probabilities?

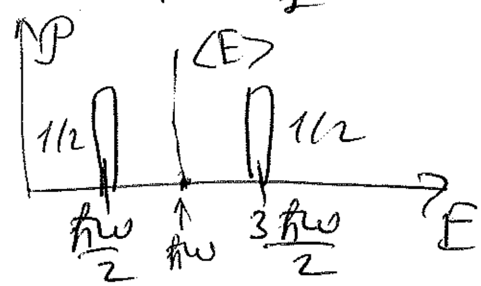
Outcomes: $E_0 = \frac{\hbar\omega}{2}$ with $P_0 = |C_0|^2 = \frac{1}{2}$

$E_1 = \frac{3}{2}\hbar\omega$ with $P_1 = |C_1|^2 = 1/2$

(b) What is the expectation value of energy?

$$\langle E \rangle = P_0 \cdot E_0 + P_1 \cdot E_1 = \frac{1}{2}\left(\frac{\hbar\omega}{2} + \frac{3\hbar\omega}{2}\right) = \hbar\omega$$

Sanity check:



(c) **If you have time:** what is the time-evolved state $\Psi(x,t)$? Does the energy expectation value change with time?

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}E_0 t}\varphi_0(x) + e^{-\frac{i}{\hbar}E_1 t}\varphi_1(x)\right) =$$

$$= \frac{1}{\sqrt{2}}e^{-\frac{i\omega t}{2}}\left(\varphi_0(x) + e^{-i\omega t}\varphi_1(x)\right)$$

Clearly, $|C_{0,1}(t)|^2 = |C_{0,1}(0)|^2 \Rightarrow \langle E \rangle$ doesn't depend on time

as expected from t-independ. $\hat{H}$