Phys 451 Winter 2026

Worksheet #6

(Friday, January 16, 2026)

Name

Questions (5 pts):

Express momentum (and position, if you get to (b)) operators in terms of raising and lowering operators and use the number representation to calculate:

(a) Expectation value of the momentum operator when the quantum harmonic oscillator is an energy eigenstate |n>, i.e.

$\langle\hat{P}\rangle = \langle n|\hat{P}|n\rangle = i\sqrt{\frac{\hbar m\omega}{2}}\,\langle n|a^+ - a|n\rangle =$

$a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$ $\quad a|n\rangle = \sqrt{n}\,|n-1\rangle$

$\hat{P} = i\sqrt{\frac{\hbar m\omega}{2}}\,(a^+ - a)$

$$= i\sqrt{\frac{\hbar m\omega}{2}}\left(\sqrt{n+1}\,\underbrace{\langle n|n+1\rangle}_{=0} - \sqrt{n}\,\underbrace{\langle n|n-1\rangle}_{=0}\right) = 0$$

(b) **If you have time:** expectation value of position operator-squared in an energy eigenstate |n>, i.e.

$\langle X^2\rangle = \langle n|X^2|n\rangle = \frac{\hbar}{2m\omega}\langle n|a^{+2} + a^2 + a^+a + aa^+|n\rangle =$

$\langle n\pm 2|n\rangle = 0$

$\hat{X} = \sqrt{\frac{\hbar}{2m\omega}}\,(a^+ + a)$

$a^2|n\rangle \Rightarrow |n-2\rangle$

$a^{+2}|n\rangle \Rightarrow |n+2\rangle$

$$= \frac{\hbar}{2m\omega}\langle n|\underbrace{a^+a}_{N} + \underbrace{aa^+}_{N+1}|n\rangle = \frac{\hbar}{2m\omega}(2n+1)$$