

Worksheet #5

(Wednesday, January 14, 2026)

Name

Question (5 pts):

Consider the number operator $N = a^\dagger a$, where $a^\dagger$ and a are creation and annihilation operators, respectively.

(a) Find the commutator $[N, a^\dagger]$

$$\begin{aligned}
 [\hat{N}, a^\dagger] &= [a^\dagger a, a^\dagger] = a^\dagger [a, a^\dagger] + \underbrace{[a^\dagger, a^\dagger]}_{=0} a = \\
 &= a^\dagger [a, a^\dagger] = \underbrace{a^\dagger}_{\substack{\text{"1"} \\ \leftarrow \text{WS\#4}}}
 \end{aligned}$$

$[\hat{A}\hat{B}\hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}$
 $\uparrow \quad \uparrow \quad \uparrow$
 $a^\dagger \quad a \quad a^\dagger$

(b) **If you have time:** use properties of the number operator N to find the following:

$$\begin{aligned}
 \underbrace{\langle n | a^\dagger a | n \rangle}_{\substack{\text{"N"} \\ \leftarrow \hat{N}}} &= \langle n | \underbrace{\hat{N}}_{\substack{\text{"N"} \\ \leftarrow \hat{N}}} | n \rangle = \langle n | n | n \rangle = \underbrace{n}_{\substack{\text{"n/n"} \\ \leftarrow \langle n | n \rangle = 1}}
 \end{aligned}$$

$$\begin{aligned}
 \underbrace{\langle n | a a^\dagger | n \rangle}_{\substack{\text{"N"} \\ \leftarrow \hat{N}+1}} &= \langle n | \hat{N} + 1 | n \rangle = \underbrace{n+1}_{\leftarrow \text{WS\#4}}
 \end{aligned}$$