Worksheet #4

(Monday, January 12, 2026)

Name

Questions (5 pts):

Consider the number operator N = $a^\dagger a$, where $a^\dagger$ and a are creation and annihilation operators, respectively.

On the board

(a) Is $a^\dagger a = aa^\dagger$? In other words, find the commutator $[a,a^\dagger]$

$$a^+a \Rightarrow u^2+v^2+i[u,v] \; ; \; aa^+ = u^2+v^2-i[u,v]$$

$$\underbrace{[a,a^+]}_{\parallel} = -2i[u,v] = -2i\cdot\frac{m\omega}{2\hbar}\cdot\frac{1}{m\omega}\underbrace{[\hat{X},\hat{P}]}_{i\hbar} = \underset{\sim}{1}$$

$$aa^+ - \underbrace{a^+a}_{\hat{N}}$$

(b) Using the result of (a), express $aa^\dagger$ in terms of the number operator N

$$aa^+ = a^+a + 1 = \hat{N} + 1$$

$\hat{N}$

(c) **If you have time:** Find the relationship between the Hamiltonian of the 1D harmonic oscillator and the number operator.

$$\overbrace{a^+ a}^{\hat{N}} = u^2 + v^2 + i\,[u, v] =$$

$$= \frac{m\omega}{2\hbar}\left(\hat{X}^2 + \frac{\hat{p}^2}{m^2\omega^2} + i\left[\hat{X}, \frac{\hat{p}}{m\omega}\right]\right) =$$

$$= \frac{\hat{p}^2}{2\hbar m\omega} + \frac{m\omega}{2\hbar}\hat{X}^2 + \underbrace{\frac{1}{2\hbar}\cdot i \cdot i\hbar}_{-\frac{1}{2}} = \frac{\hat{H}}{\hbar\omega} - \frac{1}{2}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{X}^2}{2}$$

So, $\hat{H} = \hbar\omega\left(\hat{N} + \frac{1}{2}\right)$