PH 451 **Winter 2026**

Worksheet #2

Wednesday, January 7, 2026

Name:

Questions (5 pts):

Consider a particle in an infinite potential well (V = 0 for 0<x<L and V = ∞ elsewhere). As you know, the eigenfunctions of the Hamiltonian for this system are given by

$$\varphi_n(x) = \sqrt{\frac{2}{L}} sin\left(\frac{\pi n x}{L}\right)$$

It is known that the particle is in a state described by some wave function ψ(x).

Write down the integrals you would need to take to calculate the following:

(a) The probability of finding the particle somewhere between 0 and L/2

$$\int_0^{L/2} |\psi(x)|^2 dx$$

(b) The coefficients c_n if you present ψ(x) as a superposition of eigenfunctions $\varphi_n(x)$, i.e. $\psi(x) = \sum_n c_n \varphi_n(x)$

$$c_n = \int_0^L \varphi_n^*(x)\psi(x)dx = \sqrt{\frac{2}{L}}\int_0^L \sin\frac{\pi n x}{L}\,\psi(x)dx$$

(c) **If you have time:** You make a measurement of energy. What are the possible outcomes of the measurement? How do you decide which outcomes are likely or unlikely?

$$E_n = \frac{\hbar^2\pi^2 n^2}{2mL^2}$$

$$P_n = |c_n|^2$$

(if 0 → not realized)

Analysis of WS#2

Common mistakes:

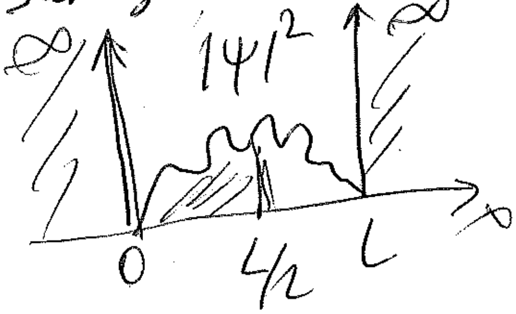
(a)

- prob. to find the particle in $[0, \frac{L}{2}] \Rightarrow$

- $\left|\int_0^{L/2} |\Psi|^2 dx\right|^2$ instead of $\int_0^{L/2} |\Psi|^2 dx$

sanity check: remember $\int_0^L |\Psi|^2 dx = 1$

total prob. = 1 (total area under $|\Psi|^2$ curve)



$\Psi(x)$ where is the particle?

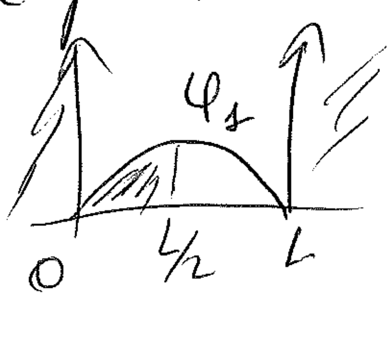
answers this

$\int_0^{L/2} |\Psi|^2 dx \Leftarrow$ a part of the total prob

Prob. to find the particle from 0 to L/2

- $\int_0^{L/2} |\varphi_n(x)|^2 dx \leftarrow$ this would be true if the particle was in an eigenstate $\varphi_n(x)$ not in $\Psi(x)$

Ex.



(b) If $\Psi(x) = \sum_n C_n \varphi_n(x)$, what is C_n?

$\times\, \varphi_m^*(x)$, $\int \Rightarrow$

$$\int_0^L \varphi_m^*(x)\Psi(x)\,dx = \sum_n C_n \underbrace{\int_0^L \varphi_m^* \varphi_n\,dx}_{=\delta_{mn}} = C_m$$

So $C_n = \int_0^L \varphi_n^*(x)\Psi(x)dx = \sqrt{\frac{2}{L}}\int_0^L \sin\frac{\pi n x}{L}\,\Psi(x)dx$

Remember that $\varphi_n = \sqrt{\frac{2}{L}}\sin\frac{\pi n x}{L}$ only on $[0, L]$

so can't have $\int_{-\infty}^{+\infty} \sin\frac{\pi n x}{L}\,\Psi(x)dx$

(c) outcomes of energy measurement $\Rightarrow E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$

$$P_n = |C_n|^2 = \frac{2}{L}\left|\int_0^L \sin\frac{\pi n x}{L}\,\Psi(x)dx\right|^2$$

note that $|\int|^2$ shows up only when we measure some quantity which requires presenting $\Psi(x)$ as superposition of eigenstates of the observable we are measuring! Like energy!

($|\int|^2$ does <u>not</u> show up when we ask <u>where</u> the particle is)