PH ~~451~~/551 **Winter 2026**

Worksheet # 18

(Friday, February 27, 2026)

Name

Questions (5 pts):

(a) Calculate the total fine-structure energy corrections (in terms of $\alpha^4 mc^2$) to the n = 1 and n = 2 energy levels of the hydrogen atom.

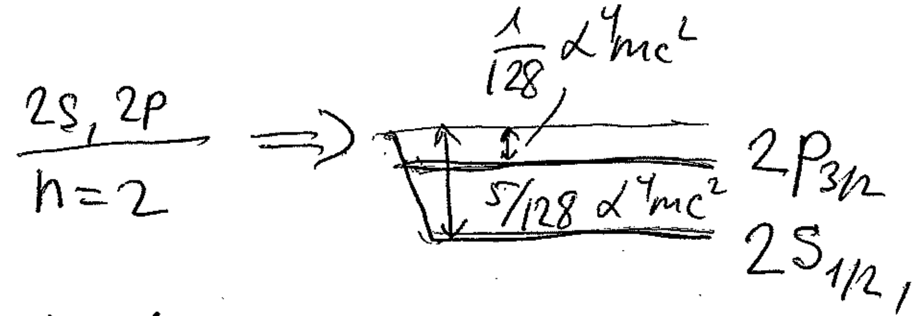
$$E^{(1)}_{fine} = -\frac{1}{2}\alpha^4 mc^2 \frac{1}{n^3}\left[\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right]$$

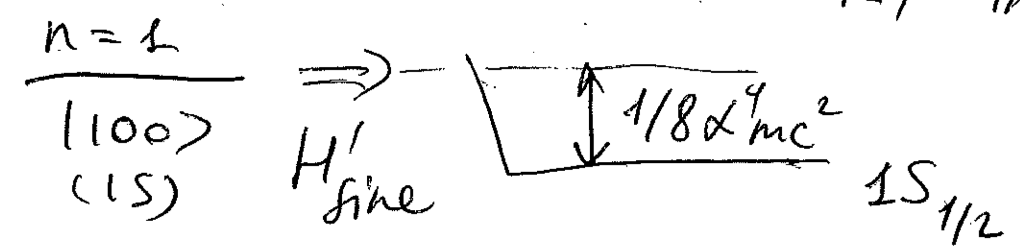
$n = 1 \Rightarrow l = 0,\ s = 1/2 \Rightarrow j = 1/2 \Rightarrow E^{(1)}_{fine} = -\frac{1}{2}\alpha^4 mc^2 \cdot 1 \cdot$
$|100\rangle$
$\cdot \left[\frac{1}{1/2+1/2} - \frac{3}{4}\right] = -\frac{1}{8}\alpha^4 mc^2$

$n = 2 \Rightarrow l = 0,\ s = 1/2 \Rightarrow j = 1/2 \Rightarrow E^{(1)}_{fine} = -\frac{1}{2}\alpha^4 mc^2 \cdot \frac{1}{8} \cdot$
$|200\rangle$ $\cdot \left(\frac{1}{1} - \frac{3}{8}\right) = -\frac{5}{128}\alpha^4 mc^2$
$|21m\rangle$ $l = 1,\ s = 1/2 \Rightarrow j = 3/2, 1/2 \Rightarrow$ for $j = 1/2$ same $E^{(1)}_{fine}$

(b) Sketch the energy level diagram and indicate the energy corrections and corresponding states in spectroscopic notations (which include information on n, *l*, and j quantum numbers)

for $j = 3/2$:
$E^{(1)}_{fine} = -\frac{1}{2}\alpha^4 mc^2 \cdot \frac{1}{8} \cdot \left(\frac{1}{2} - \frac{3}{8}\right) = -\frac{1}{128}\alpha^4 mc^2$





(c) What are the relative contributions of H'_{rel}, H'_{SO}, and H'_D to the overall correction for the n = 1 level?

For $n=1 \Rightarrow \underbrace{H'_{SO}}_{\sim \vec{L}\cdot\vec{S}} \rightarrow E^{(1)}_{SO} = 0$, so only H'_{rel} & H'_D contribute

1s or $|100\rangle$

$$E^{(1)}_{rel} = -\frac{1}{2}\alpha^4 mc^2\left[\frac{1}{n^3(l+\frac{1}{2})} - \frac{3}{4n^4}\right]$$

$$n=1,\ l=0 \Rightarrow E^{(1)}_{rel} = -\frac{1}{2}\alpha^4 mc^2\left[\frac{1}{1/2} - \frac{3}{4}\right] = -\frac{5}{8}\alpha^4 mc^2$$

From (a): $E^{(1)}_{fine} = -\frac{1}{8}\alpha^4 mc^2 \Rightarrow E^{(1)}_D = \frac{1}{2}\alpha^4 mc^2$