PH 551 **Winter 2026**

Worksheet # 16

(Friday, February 20, 2026)

Name

Questions (5 pts):

(a) Use the table of Clebsch-Gordan coefficients for the addition of $j_1 = 1$ and $j_2 = ½$ to present the state $|½ ½>$ in the coupled basis in terms of states in the uncoupled basis. Label all the quantum numbers in your expression (i.e. J, M, j_1, j_2, m_1, m_2).

Table 11.3 $\Rightarrow$ $\overset{J\ M}{\left|\tfrac{1}{2}\tfrac{1}{2}\right\rangle} = \sqrt{\frac{2}{3}}\,\overset{j_1\ j_2\ m_1\ m_2}{\left|1\ \tfrac{1}{2}\ 1\ -\tfrac{1}{2}\right\rangle} -$

$- \frac{1}{\sqrt{3}}\,\overset{j_1\ j_2\ m_1\ m_2}{\left|1\ \tfrac{1}{2}\ 0\ \tfrac{1}{2}\right\rangle}$; sanity check: $m_1 + m_2 = M$

$\underbrace{|j_1 - j_2|}_{=1/2} \le J = \frac{1}{2} \le \underbrace{j_1 + j_2}_{3/2}$

(b) use the result of (a) to solve the following problem.

An electron in a p-state ($l = 1$) has a total angular momentum (**J** = **L**+**S**) of $J = ½$. It is also known that the total angular momentum has a maximal possible positive projection on the z-axis. What is the probability to find the electron in the spin-down configuration?

$l = 1 \leftarrow j_1$ $\quad s = 1/2 \leftarrow j_2$ $\quad J = 1/2$

$M = 1/2$

Need: $m_s = -\frac{1}{2} \leftarrow m_2$

$\Downarrow$

the probability that e^- is in $\overset{j_1\ j_2\ m_1\ m_2}{\underset{l\ s\ m_l\ m_s}{\left|1\ \tfrac{1}{2}\ 1\ -\tfrac{1}{2}\right\rangle}}$

from (a) is $\boxed{\frac{2}{3}}$