Worksheet # 15

(Wednesday, February 18, 2026)

Name

Questions (5 pts):

An electron in the hydrogen atom is in a p-state (i.e. the orbital quantum number $l = 1$). The total angular momentum **J = L + S**.

(a) Apply the rules of addition of angular momenta to find the possible values of the quantum number for the total angular momentum J.

$$l = 1, \quad s = 1/2 \implies J = l + s, \dots |l - s| \implies$$

$$J = \frac{3}{2}, \frac{1}{2}$$

(b) Write down the possible states |J M>.

$$J = \frac{3}{2} \implies M = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$$

$$J = \frac{1}{2} \implies M = \frac{1}{2}, -\frac{1}{2}$$

$$|\tfrac{1}{2}\,\tfrac{1}{2}\rangle, |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$$

$$\text{So:} \quad |\tfrac{3}{2}\,\tfrac{3}{2}\rangle; |\tfrac{3}{2}\,\tfrac{1}{2}\rangle; |\tfrac{3}{2}\,-\tfrac{1}{2}\rangle; |\tfrac{3}{2}\,-\tfrac{3}{2}\rangle;$$

(c) If you have time: how many states |J M> are there ? Compare this number to the number of states in the corresponding uncoupled basis |l m_l s m_s> and comment.

$$\underline{6} \; |JM\rangle \text{ states}$$

$$\text{Now } |l\,m_e\,s\,m_s\rangle:$$

$$|l\,m_e\rangle: \quad \left.\begin{array}{l} |1\;1\rangle \\ |1\;0\rangle \\ |1\;-1\rangle \end{array}\right\} 3$$

$$\text{Each of } |l\,m_e\rangle \text{ can be combined with each of } |s\,m_s\rangle \implies$$

$$|s\,m_s\rangle: \quad \left.\begin{array}{l} |\tfrac{1}{2}\;\tfrac{1}{2}\rangle \\ |\tfrac{1}{2}\;-\tfrac{1}{2}\rangle \end{array}\right\} 2$$

$$3 \times 2 = \underline{6} \qquad \text{The same dimensionality !!}$$