PH 451/551 **Winter 2026**

Worksheet # 14

(Monday, February 16, 2026)

Name

Questions (5 pts):

As we showed, the hyperfine perturbation is represented (in the uncoupled basis) by the following matrix:

$$H'_{hf} \doteq \frac{A}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(a) Diagonalize the matrix to find the energy corrections to the ground state of the H-atom due to hyperfine interactions.

$E'_{hf} = \frac{A}{4} \Rightarrow |++\rangle, |--\rangle$

Diagonalize $\frac{A}{4}\begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$: $\det\begin{pmatrix} -\frac{A}{4}-\lambda & 2\cdot\frac{A}{4} \\ \frac{A}{4}\cdot 2 & -\frac{A}{4}-\lambda \end{pmatrix} = 0$

$\left(\frac{A}{4}+\lambda\right)^2 = \left(\frac{A}{2}\right)^2 \Rightarrow \frac{A}{4}+\lambda = \pm\frac{A}{2} \Rightarrow \lambda = \frac{A}{4}; -\frac{3A}{4}$

(b) Find the eigenstates corresponding to each correction.

E'_{hf}

$|E'_{hf} = \frac{A}{4}\rangle$: $\begin{pmatrix} -\frac{A}{4}-\frac{A}{4} & \frac{A}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \Rightarrow -c_1 + c_2 = 0$

$c_1 = c_2 \Rightarrow$

$\frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle) \Leftarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|E'_{hf} = -\frac{3A}{4}\rangle$: $\begin{pmatrix} -\frac{A}{4}+\frac{3A}{4} & \frac{A}{2} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \Rightarrow c_1 = -c_2 \Rightarrow$

$\frac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$ $\Leftarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow$

(c) Sketch the energy level diagram showing the effect of the hyperfine interaction. Indicate the states.