

Worksheet #11

(Friday, January 30, 2026)

Name

Questions (5 pts):

As we showed, the second-order energy correction is given by

$$E_n^{(2)} = \langle n^{(0)} | H' | n^{(1)} \rangle, \text{ where } |n^{(1)}\rangle = \sum_{k \neq n} \frac{H'_{kn}}{E_n^{(0)} - E_k^{(0)}} |k^{(0)}\rangle, \text{ where } H'_{kn} = \langle k^{(0)} | H' | n^{(0)} \rangle.$$

- (a) Plug in the expression for $|n^{(1)}\rangle$ in the expression for the second-order energy correction and simplify the expression as much as possible.

$$E_n^{(2)} = \langle n^{(0)} | H' \sum_{k \neq n} \frac{H'_{kn}}{E_n^{(0)} - E_k^{(0)}} |k^{(0)}\rangle = \sum_{k \neq n} \frac{H'_{kn}}{E_n^{(0)} - E_k^{(0)}} \cdot \langle n^{(0)} | H' | k^{(0)} \rangle$$

$$\underbrace{\langle n^{(0)} | H' | k^{(0)} \rangle}_{H'_{nk}} = \sum_{k \neq n} \frac{|H'_{kn}|^2}{E_n^{(0)} - E_k^{(0)}}$$

$$H'_{kn} \cdot H'_{nk} = |H'_{kn}|^2$$

- (b) Based on the expressions for the second-order energy correction and first-order state correction, and given that the corrections are supposed to be "small", develop a criterion for what exactly is "small" – i.e. criterion for validity of perturbation theory.

Need to ensure $|H'_{kn}| \ll |E_n^{(0)} - E_k^{(0)}|$

———— $E_{n+1}^{(0)}$

$\underbrace{E_n^{(0)}}_{\text{energy correction}} \ll |E_{n+1}^{(0)} - E_n^{(0)}|$