Phys 451 **Winter 2026**

Worksheet #10

(Wednesday, January 28, 2026)

Name

Questions (5 pts):

As we discussed, there is a convention for how the states are normalized in the perturbation theory.

(a) Apply the convention ($|n\rangle$ is normalized, $\langle n^{(0)}|n\rangle$ is real) to the first-order approximation to obtain $\langle n^{(0)}|n^{(1)}\rangle$.

We see that $\langle n^{(0)}|n^{(1)}\rangle + \langle n^{(1)}|n^{(0)}\rangle = 0$

In general $\langle a|b\rangle = \langle b|a\rangle^* \Rightarrow$

$\langle n^{(0)}|n^{(1)}\rangle = \langle n^{(1)}|n^{(0)}\rangle^* = \langle n^{(1)}|n^{(0)}\rangle$

convention: $\langle n^{(0)}|n^{(1)}\rangle$ real

So $\langle n^{(0)}|n^{(1)}\rangle + \langle n^{(0)}|n^{(1)}\rangle = 0 \Rightarrow$

$\langle n^{(0)}|n^{(1)}\rangle = -\langle n^{(0)}|n^{(1)}\rangle \Rightarrow$ if a real number, this means $\langle n^{(0)}|n^{(1)}\rangle = 0$

(b) **If you have time:** Write down the normalization condition to the second-order approximation. Use the result of (a) to figure out what it implies for the $\langle n^{(1)}|n^{(1)}\rangle$ must have inner product.

λ^2: $|n\rangle_\lambda = |n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \lambda^2|n^{(2)}\rangle$; ${}_\lambda\langle n|n\rangle_\lambda = 1$

${}_\lambda\langle n|n\rangle_\lambda = (\langle n^{(0)}| + \lambda\langle n^{(1)}| + \lambda^2\langle n^{(2)}|)(|n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \lambda^2|n^{(2)}\rangle) = \underbrace{\langle n^{(0)}|n^{(0)}\rangle}_{=1} + \lambda\underbrace{\langle n^{(1)}|n^{(0)}\rangle}_{=0} + \lambda^2\langle n^{(2)}|n^{(0)}\rangle +$

$+ \lambda\underbrace{\langle n^{(0)}|n^{(1)}\rangle}_{=0} + \lambda^2\langle n^{(1)}|n^{(1)}\rangle + \lambda^2\langle n^{(0)}|n^{(2)}\rangle + O(\lambda^3) \Rightarrow$

$\langle n^{(2)}|n^{(0)}\rangle + \langle n^{(0)}|n^{(2)}\rangle + \langle n^{(1)}|n^{(1)}\rangle = 0 \Rightarrow$

By convention, $\langle n^{(0)} | n^{(2)} \rangle = \langle n^{(2)} | n^{(0)} \rangle$

So, $\langle n^{(1)} | n^{(1)} \rangle = -2 \langle n^{(0)} | n^{(2)} \rangle$