List of suggested review questions (based on Ch. 1-8)

POSTULATES List the postulates of quantum mechanics.

PHYSICAL OBSERVABLES For each of the following physical observables, answer these questions:

Position For the operator $\hat{x}$ characterizing the position of a particle,

- What is the action of $\hat{x}$ on a quantum wave function $\psi(x,t)$?
- In Dirac notation, what is the eigenvalue equation for the eigenvalues and eigenstates of $\hat{x}$? What are the allowed values of the eigenvalues of $\hat{x}$?
- What are the position-space eigenfunctions of $\hat{x}$ i.e. what is $\langle x|x'\rangle$?
- In both Dirac notation, and in terms of the position-space eigenfunctions, what is the completeness relation for the eigenstates of $\hat{x}$?

Momentum For the operator $\hat{p}$ characterizing the (linear) momentum of a particle,

- What is the action of $\hat{p}$ on a quantum wave function $\psi(x,t)$?
- In Dirac notation, what is the eigenvalue equation for the eigenvalues and eigenstates of $\hat{p}$? What are the allowed values of the eigenvalues of $\hat{p}$?
- What are the position-space eigenfunctions of $\hat{p}$ i.e. what is $\langle x|p\rangle$?
- In both Dirac notation, and in terms of the position-space eigenfunctions, what is the completeness relation for the eigenstates of $\hat{p}$?

Orbital angular momentum For the operator $\vec{L}$ characterizing the orbital angular momentum of a particle,

- What are the commutation relations obeyed by the components $\{L_x, L_y, L_z\}$ amongst themselves? What's the commutator between these components and $\vec{L}^2$?
- In spherical coordinates, what is the action of the operators $\vec{L}^2$ and L_z on a wave function $\psi(\vec{x},t)$?
- In Dirac notation, what is the eigenvalue equation for the eigenvalues and eigenstates of $\vec{L}^2$ and L_z? What are the allowed values of the eigenvalues of $\vec{L}^2$ and L_z?
- What are the position-space eigenfunctions of $\vec{L}^2$ and L_z?
- In both Dirac notation, and in terms of the position-space eigenfunctions, what is the completeness relation for these eigenstates?

Spin angular momentum For the operator $\vec{S}$ characterizing the spin of a particle,

- What are the commutation relations obeyed by the components $\{S_x, S_y, S_z\}$ amongst themselves? What's the commutator between these components and $\vec{S}^2$?
- In Dirac notation, what is the eigenvalue equation for the eigenvalues and eigenstates of $\vec{S}^2$ and S_z? What are the allowed values of the eigenvalues of $\vec{S}^2$ and S_z?
- In Dirac notation, what is the completeness relation for these eigenstates?
- In both Dirac notation and the corresponding matrix representations, what are the eigenstates of the operators $\{S_x, S_y, S_z\}$ expressed in the S_z-basis for the cases of spin-1/2 and spin-1?
- For each of the cases of spin-1/2 and spin-1, what are the matrix representations of the components $\{S_x, S_y, S_z\}$ of the operator $\vec{S}$ in the basis of eigenstates of S_z?

QUANTUM UNCERTAINTY For two physical observables described by operators $\hat{A}$ and $\hat{B}$,

- What is the commutator of $\hat{x}$ and $\hat{p}$?
- What is the uncertainty relation charaterizing how accurately one may simultaneously measure the values of $\hat{A}$ and $\hat{B}$? Be certain to define all terms.
- When is it possible for $\hat{A}$ and $\hat{B}$ to have a common set of eigenstates?

TIME EVOLUTION Consider a system in a quantum state $|\psi(t)\rangle$ that is subject to a Hamiltonian H with eigenstates $\{|E_n\rangle\}$ and corresponding energies $\{E_n\}$. The corresponding position-space wave function is $\psi(x,t) \equiv \langle x|\psi(t)\rangle$, and the energy eigenfunctions are $\varphi_n(x) \equiv \langle x|E_n\rangle$.

- Write down the Schrödinger Equation (SE) in both Dirac notation, and as a differential equation in position-space.
- Write down the Time-independent Schrödinger equation (TISE) in both Dirac notation, and as a differential equation in position-space.
- If the initial state is $|\psi_0\rangle$, what is $|\psi(t)\rangle$? Write down the answer in both Dirac notation, and in terms of the position-space energy eigenfunctions.

PARTICULAR PHYSICAL SYSTEMS For each of the following physical systems,

- Particle of mass m in a box of width a extending from $x = 0$ to $x = a$.
- Particle of mass μ confined to move on a ring of radius a.
- Particle with mass m, charge q, and spin-1/2 in a magnetic field $\vec{B}$.
- Electron in a Hydrogen atom.
- Free particle of mass m.

answer the following questions:

- What is the potential energy ? Sketch.
- What are the boundary conditions?
- What is the Hamiltonian for this system?
- Write down the time-independent Schrodinger equation. Write down the answer both in Dirac notation and as a differential equation in terms of the position-space energy eigenfunctions.
- What is the energy spectrum? Continuous or discrete?
- What are the position-space wavefunctions?