Perturbation theory

Read McIntyre 10.3-10.4

PH451/551, January 28, 2026

Quick check:

In the expression $H = H_0 + \lambda H'$

1. What does H_0 represent?

1. What does H' represent?

2. What does λ represent?

Quick check:

In the expression $H = H_0 + \lambda H'$

1. What does H_0 represent?
   The "original Hamiltonian" of some system whose solution is known.
2. What does H' represent?
   The "perturbation Hamiltonian" – a small term relative to H_0 .
3. What does λ represent?
   It is a bookkeeping device that keeps track of the order of the small quantities.

From last time:

Need to solve: $H|n> = En\,|n>$ *where* $H = H_0 + \lambda H'$

Energy: $E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots$

State: $|n\rangle = |n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \ldots$

The solution to the unperturbed problem is known:

$$H_0|n^{(0)}> = E_n^{(0)}\,|n^{(0)}>$$

$\langle k^{(0)}|n^{(0)}\rangle = \delta_{kn}$ - unperturbed states are orthogonal and form a complete basis: $\sum_n |n^{(0)}\rangle\langle n^{(0)}| = 1$

Consider non-degenerate case: all $E_n^{(0)}$ are different !

Find first- and second-order energy corrections and first-order state correction

From last time:

Collect terms which have the same perturbation order λ, in this case λ^1 :

$$\left(H_0 - E_n^{(0)}\right)|n^{(1)}\rangle = \left(E_n^{(1)} - H'\right)|n^{(0)}\rangle$$

Multiply by $\langle n^{(0)}|$:

$$\langle n^{(0)}|\left(H_0 - E_n^{(0)}\right)|n^{(1)}\rangle = \langle n^{(0)}|\left(E_n^{(1)} - H'\right)|n^{(0)}\rangle$$

WS 9: evaluate the L.H.S. of the equation above

Note: operator can act backwards

1. If

$$H_0\left|n^{(0)}\right\rangle = E_n^{(0)}\left|n^{(0)}\right\rangle$$

2. Then always true that

$$\left\langle n^{(0)}\right| H_0^{\dagger} = \left\langle n^{(0)}\right| E_n^{(0)*}$$

3. And for Hermitian operator

$$\left\langle n^{(0)}\right| H_0 = \left\langle n^{(0)}\right| E_n^{(0)}$$

$$\left\langle n^{(0)} \right| \left(E_n^{(0)} - E_n^{(0)} \right) \left| n^{(1)} \right\rangle = E_n^{(1)} - \left\langle n^{(0)} \right| H' \left| n^{(0)} \right\rangle$$

L.H.S. = 0, so

$$E_n^{(1)} = \left\langle n^{(0)} \right| H' \left| n^{(0)} \right\rangle$$

First-order energy correction = expectation value of the perturbation with respect to unperturbed states

Derivation – 1st order state

1. First order equation: $\neq n$

$$\left(H_0 - E_n^{(0)}\right)\left|n^{(1)}\right\rangle = \left(E_n^{(1)} - H'\right)\left|n^{(0)}\right\rangle$$

$$\left\langle m^{(0)}\right|\left(H_0 - E_n^{(0)}\right)\left|n^{(1)}\right\rangle = \left\langle m^{(0)}\right|\left(E_n^{(1)} - H'\right)\left|n^{(0)}\right\rangle$$

$$\left\langle m^{(0)}\right|\left(E_m^{(0)} - E_n^{(0)}\right)\left|n^{(1)}\right\rangle = -\left\langle m^{(0)}\right| H' \left|n^{(0)}\right\rangle$$

assume

$$\left|n^{(1)}\right\rangle = \sum_{p\neq n} c_{np}\left|p^{(0)}\right\rangle$$

Derivation – 1st order state

$$\left| n^{(1)} \right\rangle = \sum_{p \neq n} c_{np} \left| p^{(0)} \right\rangle$$

This says: I can write the "correction" to the nth state as a superposition of unperturbed states. Finding the correction is equivalent to finding the c_{np} values (I know the unperturbed states).