Applications of the Quantum Harmonic Oscillator

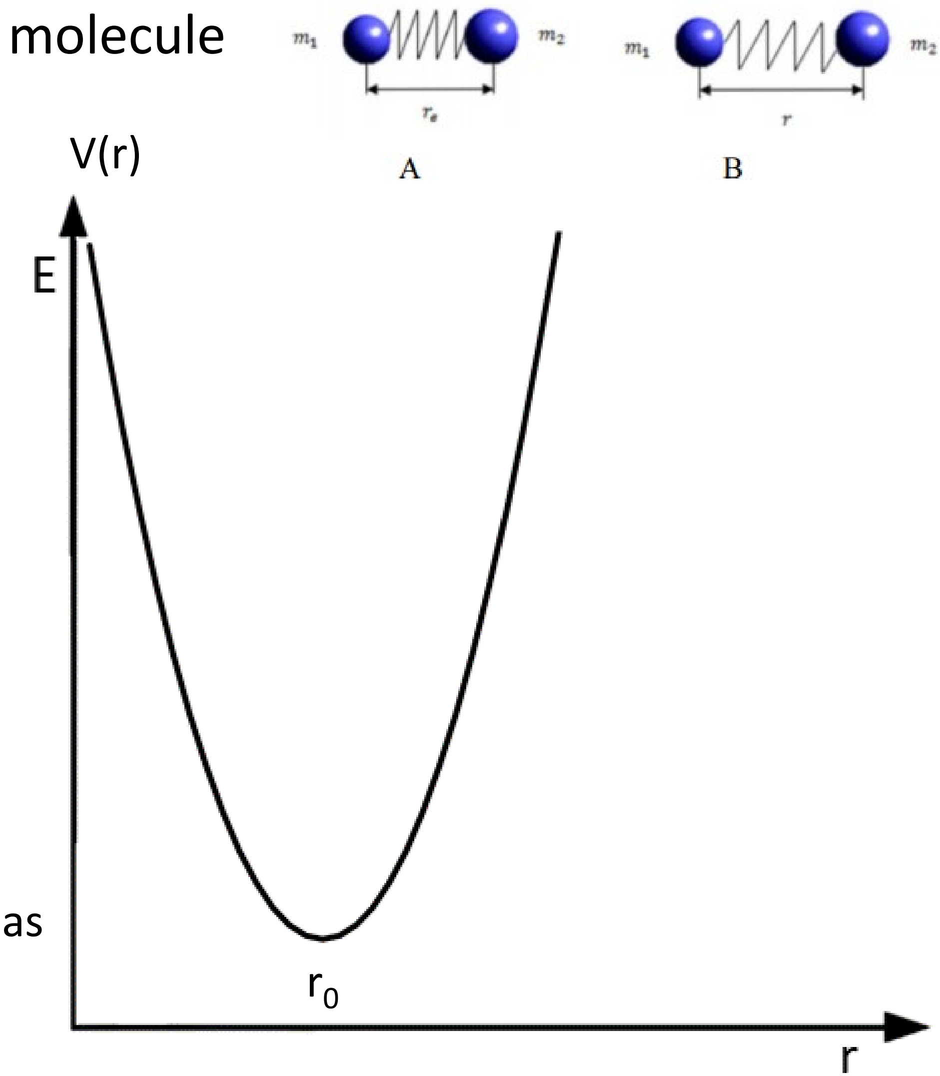
PH 451/551

January 23, 2026

McIntyre Ch. 9.9

Application: molecular spectroscopy

Potential energy for a diatomic molecule



Morse potential:

$V(r) = D_e\,(e^{-2a(r-r0)} - 2e^{-a(r-r0)})$

$a = \omega\sqrt{(\mu/2D_e)}$

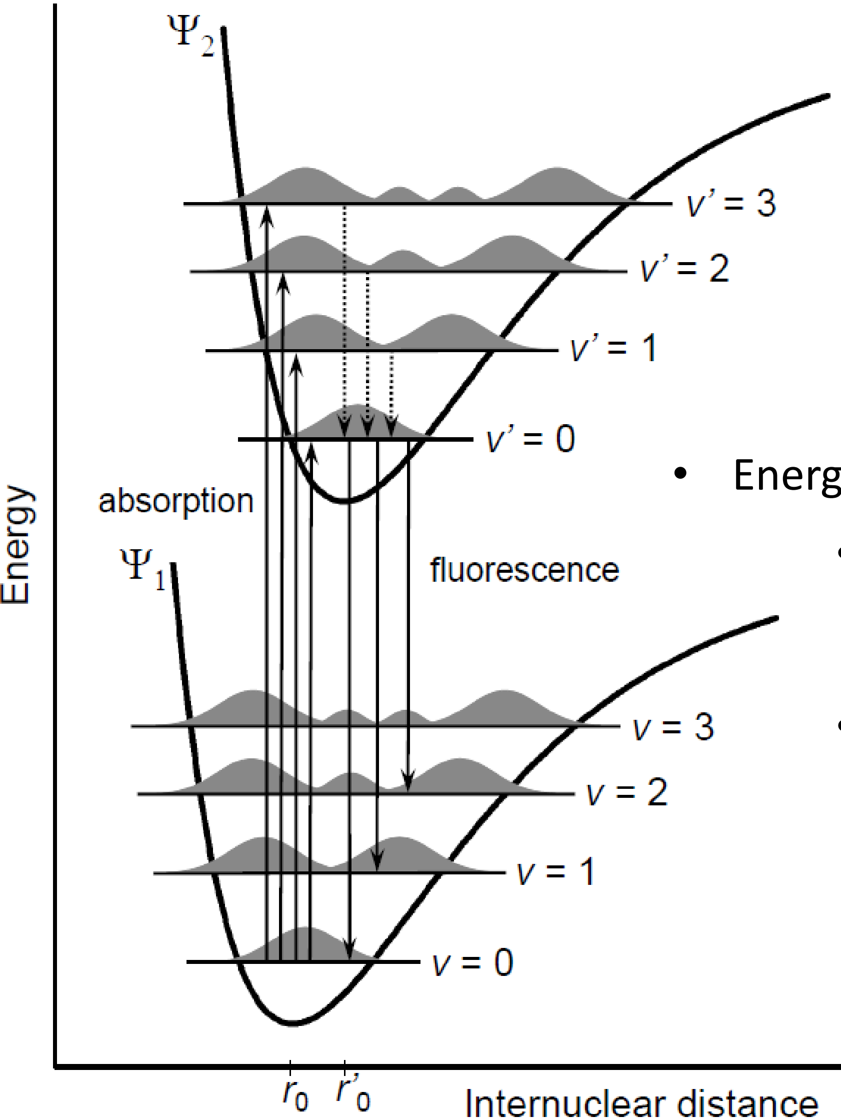
Taylor-expand V(r):

$V(r) \approx -D_e + ½\, \mu\omega^2 (r-r_0)^2 - D_e a^3\, (r-r_0)^3 + \ldots$

- Near $r = r_0$ behaves like **harmonic oscillator**
- At larger $r-r_0$ develop **anharmonicity** – use higher-order terms in Taylor expansion as a perturbation to the H.O. – we will do this using the **perturbation theory** next week !

https://specac.com/theory-articles/molecular-vibrations-explained-animated-guides/

Molecular absorption and fluorescence



- Molecular wave function:

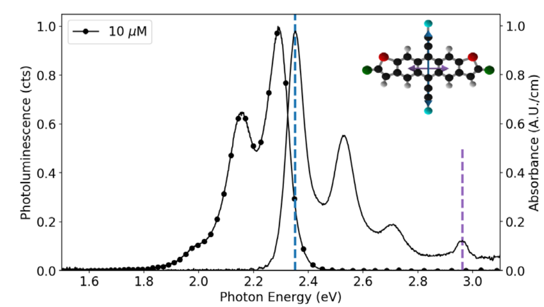
$$\Psi(\mathbf{r},\mathbf{R}) = \psi(\mathbf{r})\,\chi(\mathbf{R})$$

Electronic part

Nuclear part (Harmonic Oscillator Wave functions !)

- Energy of the molecule = electronic + vibrational + rotational
  - Strength of absorption and emission is proportional to the transition dipole moment-squared
  - Transition dipole moment relies on overlap of nuclear parts of the wave function:

$$\boldsymbol{\mu}_{12} = \int \chi^*_{2v'}\chi_{10}\,dR \int \psi^*_2 \boldsymbol{\mu}\, \psi_1\, dr$$

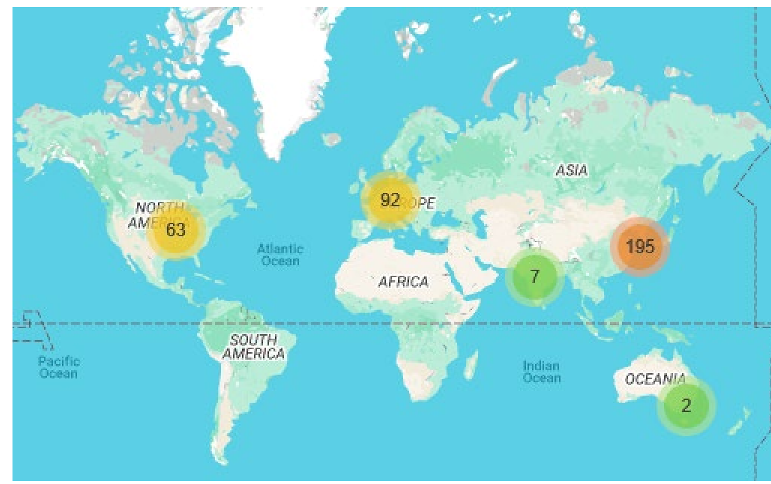


Design molecules with optical properties on demand !

OLEDs: current status

www.oled.com

https://pid.samsungdisplay.com/en/digital-signage/transparent-displays



www.oled-info.com

Current market ($38B in 2021, $350B in 2034):

- Displays (TVs, phones, watches, wearables, automotive)
- Lighting (low power consumption, brightness, color quality)

Samsung, LG, Sony, Apple, Nokia, Panasonic,…

OLEDs rely on molecular emission properties !

Organic solar cells

Power conversion efficiency:

2000: 1%

2025: 21%

Chen et al., *Nat. Mat*. **24**, 444 (2025)

Current market: $100M in 2021, $800M in 2030

Cost effectiveness for commercialization: ~10-15%

514 m²
Total area

22.5 kWp
Total capacity

378
Total films

La Rochelle, France
Installation location

Heliatek, ARMOR, infinityPV ApS, Solarmer Energy,Inc.

http://www.heliatek.com/

Organic solar cells rely on molecular absorption properties !

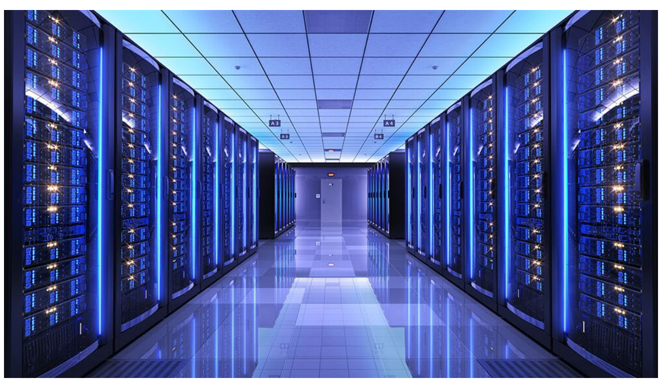
Application: spin waves

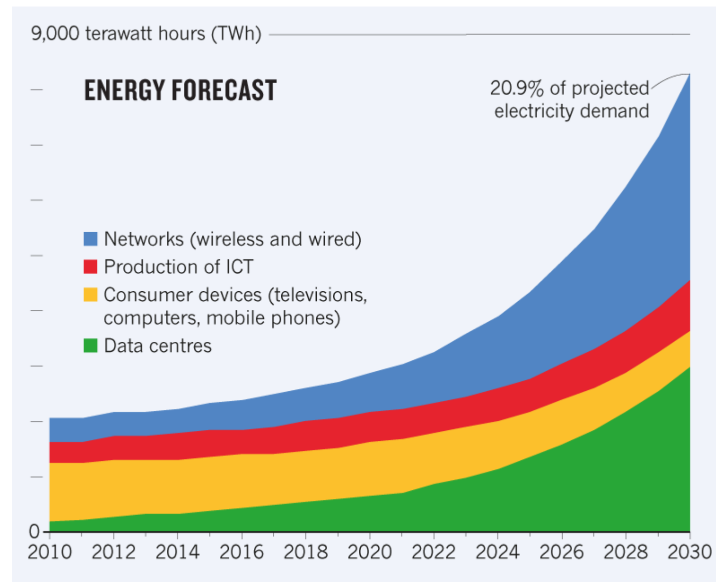
OSU Oregon State University — Computing and the environment

Energy-intensive applications - AI and cryptocurrency - are growing rapidly

Limitations of transistor-based computing

- Scaling
- Cooling



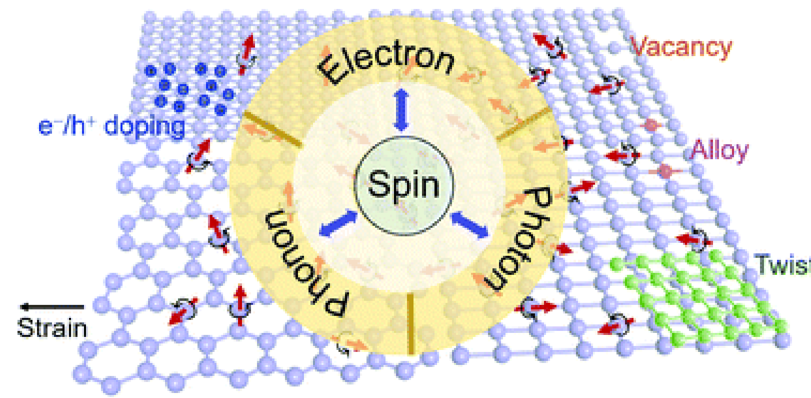


https://www.akcp.com/blog/the-real-amount-of-energy-a-data-center-use/

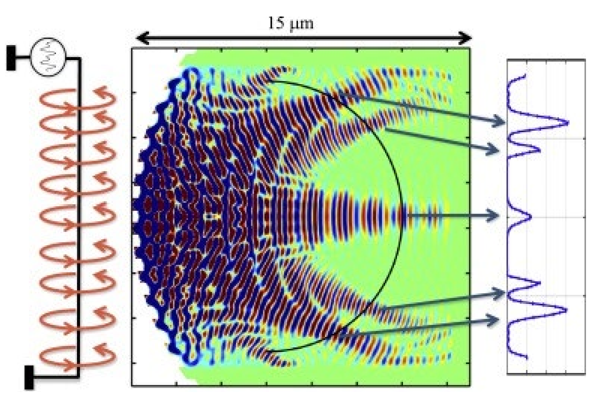
A way forward

Computing with magnets - spintronics

- No resistive heating
- Simple device architectures
- Nanoscale wave computing
- Potential for quantum transduction



Chumak et. al., *Nat. Phys*. **11** (2015)



Csaba et. al., *Phys. Lett. A* **381.** (2017).

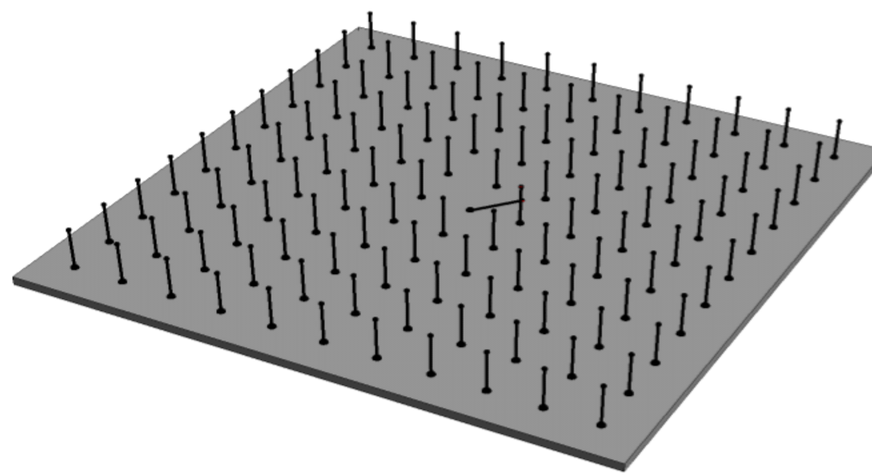
2D magnetic materials

- Many knobs for control
- Integration in nanoscale technology
- Rich fundamental physics

https://www.mouser.com/new/everspin-technologies/everspinmram/

What is a spin wave?

- Classical picture: a propagating disturbance to the equilibrium angle of magnetic moments
- Quantum picture (magnons): A **collective excitation** to a magnetic system with quantized energy



https://en.wikipedia.org/wiki/Spin_wave

SW Computing - Figures of Merit

- Gilbert damping parameter α
  - Prevent information loss in amplitude-based SW circuits
- Coherence length and coherence time
  - Governs maximum size of circuits
- Group velocity v_g
  - Governs maximum size of circuits
  - Derivative of dispersion relation $\omega(k)$

Eigenstates: Magnons!

Assume each spin has Z neighbors →

$$\mathcal{H} = -2\mathcal{J}NZs^2 - g\mu_B BNs + \mathcal{H}_0$$

$$\mathcal{H}_0 = \sum_{\boldsymbol{k}} \hat{n}_{\boldsymbol{k}} \hbar\omega_{\boldsymbol{k}}, \quad \hat{n}_{\boldsymbol{k}} = a_{\boldsymbol{k}}^\dagger a_{\boldsymbol{k}}$$

$$\boxed{\hbar\omega_{\boldsymbol{k}} = 4\mathcal{J}sZ(1-\gamma_{\boldsymbol{k}}) + g\mu_B B}$$

$$\gamma_{\boldsymbol{k}} = \frac{1}{Z}\sum_{\boldsymbol{\delta}} e^{i\boldsymbol{k}\cdot\boldsymbol{\delta}}$$

Magnons are harmonic oscillators
+ spin-spin interactions + interactions
with magnetic field

Applications of coherent states of harmonic oscillator: quantum optics

Nobel Prize in Physics 2005

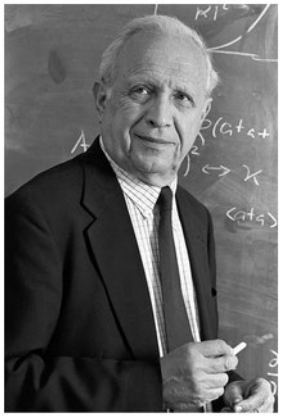


Photo: J.Reed
Roy J. Glauber
Prize share: 1/2

Photo: Sears.P.Studio
John L. Hall
Prize share: 1/4

Photo: F.M. Schmidt
Theodor W. Hänsch
Prize share: 1/4

www.nobelprize.org

The Nobel Prize in Physics 2005 was divided, one half awarded to Roy J. Glauber "for his contribution to the quantum theory of optical coherence", the other half jointly to John L. Hall and Theodor W. Hänsch "for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"

Glauber provided formal foundation for coherent states of light

Glauber founded the field of **quantum optics** – based on photons (laser fields) presented as coherent states of harmonic oscillator !

(see Glauber's Nobel paper on the class website)

Definition of a Coherent State

- Defined as eigenstates of annihilation operator: $a|\alpha\rangle = \alpha|\alpha\rangle$
- α is a complex number
- Not energy eigenstates

See McIntyre Ch. 9.8 and challenge problem in HW3 (McIntyre 9.20)

Time Evolution of Coherent States

- Time evolution preserves coherent-state form
- $\alpha(t) = \alpha(0)\, e^{(-i\omega t)}$
- Wavepacket oscillates without spreading
- Minimal uncertainty

Classical Correspondence

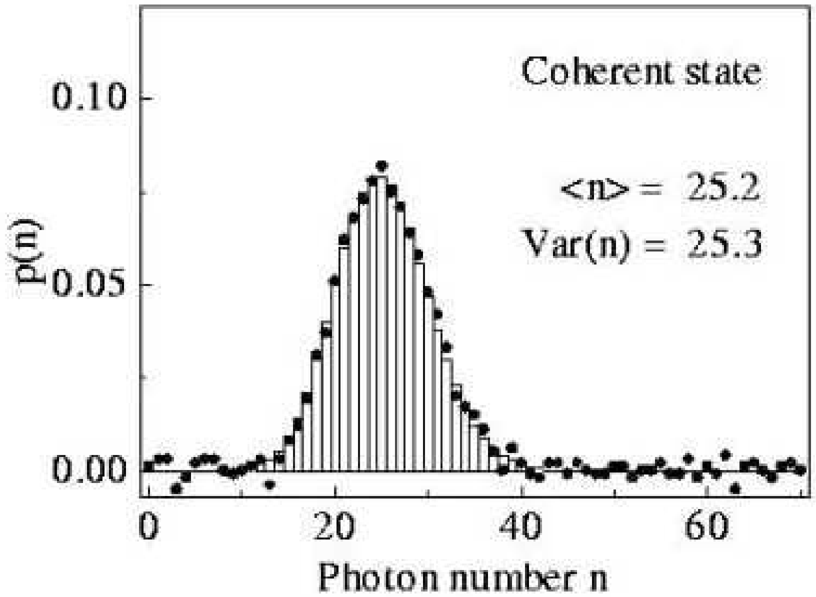
- Expectation values follow classical equations of motion
- Best quantum analog of a classical harmonic oscillator
- Bridges the gap between classical and quantum description of light

- Widely used in quantum optics:
  - Single mode of the electromagnetic field ≡ quantum harmonic oscillator
  - Laser light is approximated by coherent states (distinct from incoherent light like black-body radiation !)
  - Photon creation/annihilation operators ⟷ $a^\dagger$, a

Expansion in Number Basis

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

- Poissonian distribution in n
- Mean occupation number: $\langle n \rangle = |\alpha|^2$



The probability of detecting n photons, the photon number distribution, of the coherent state.

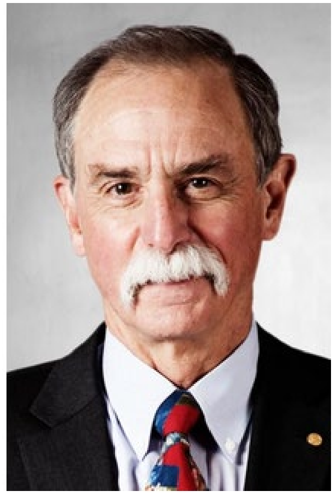
As is necessary for a Poissonian distribution the mean photon number is equal to the variance of the photon number distribution.

https://en.wikipedia.org/wiki/Coherent_state

Applications of coherent states of harmonic oscillator: ion trapping for quantum computing

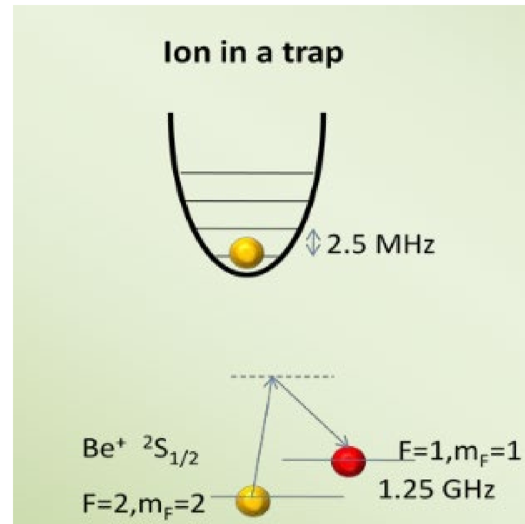
Serge Haroche

Prize share: 1/2



David J. Wineland

Prize share: 1/2



The Nobel Prize in Physics 2012 was awarded jointly to Serge Haroche and David J. Wineland "for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"

Monroe, C. R. and Wineland, D. J. (2008) Quantum Computing with Ions, *Scientific American*, August.

Wineland's Trapped Ion Oscillators

- Quantum control of single ions in harmonic traps
- Ion motion ≡ quantum harmonic oscillator
- Laser cooling prepares near-ground states
- Displacement operations generate coherent motional states

Significance of coherent states

- Benchmark for classical vs. quantum behavior
- Central to precision measurement and quantum metrology