Hydrogen atom: beyond Coulomb interaction. Angular momentum.

Read McIntyre 11.1-11.3

PH451/551

Friday, February 6, 2026

Recall: Stark effect in Hydrogen

Perturbation (n=2 subspace): need different l, need same m_l

$$H' \doteq \begin{pmatrix} 0 & -3e\mathrm{E}a_0 & 0 & 0 \\ -3e\mathrm{E}a_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} 200 \\ 210 \\ 211 \\ 21,-1 \end{matrix}$$

States:

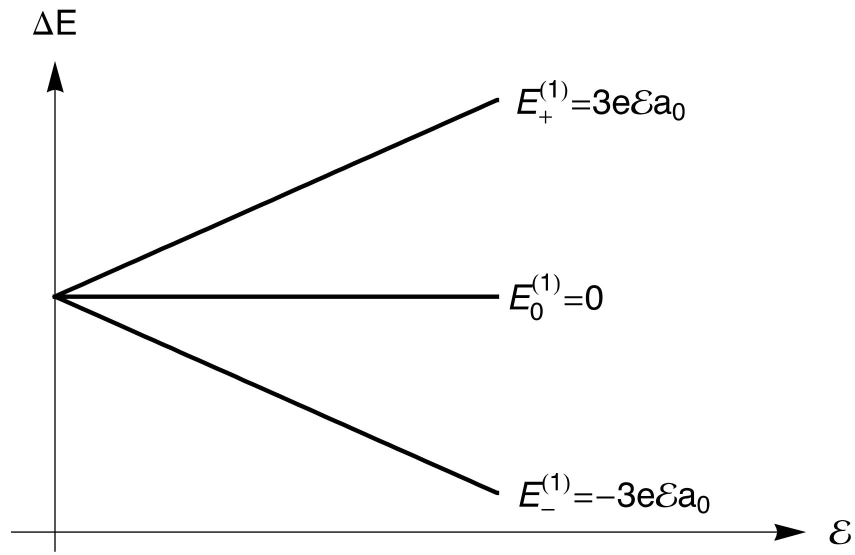
$$E = +3e\mathrm{E}a_0, -3e\mathrm{E}a_0, 0, 0$$

$$|\psi_+\rangle = \tfrac{1}{\sqrt{2}}\left[|200\rangle - |210\rangle\right]$$

$$|\psi_-\rangle = \tfrac{1}{\sqrt{2}}\left[|200\rangle + |210\rangle\right]$$

$$|\psi_3\rangle = |211\rangle$$

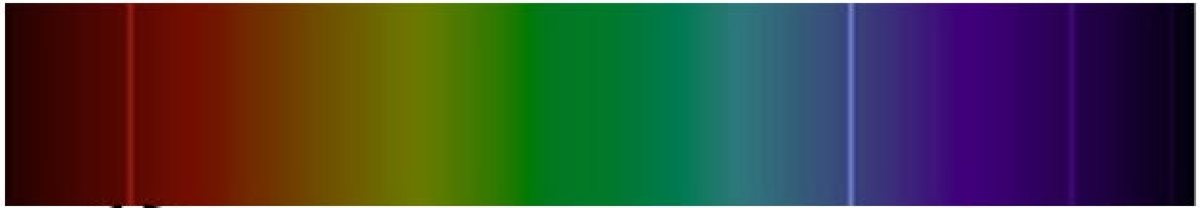
$$|\psi_4\rangle = |21,-1\rangle$$

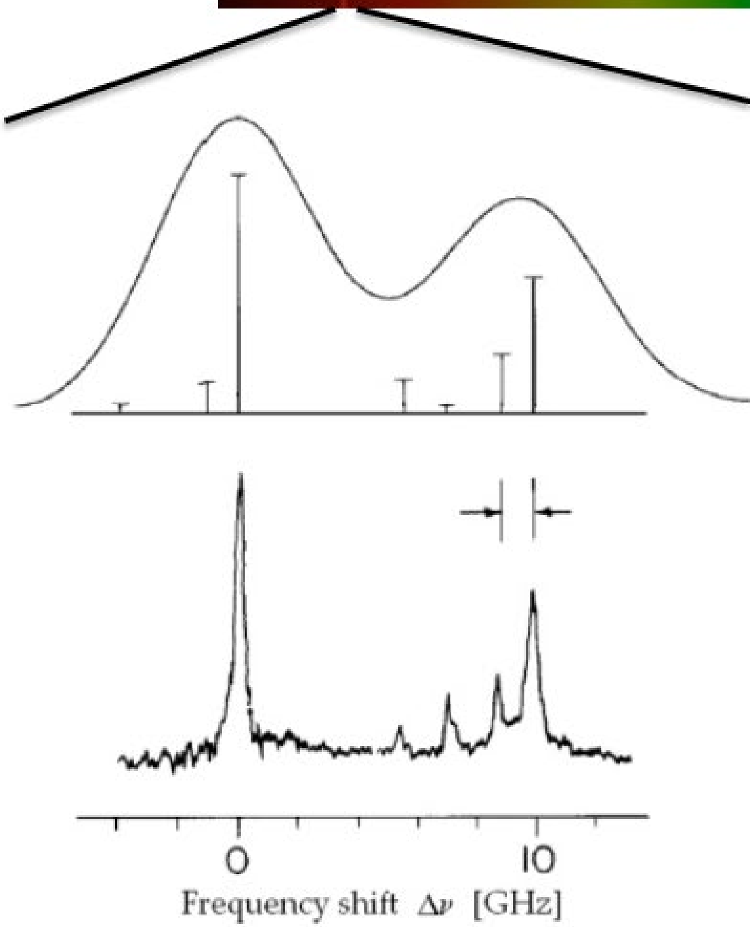


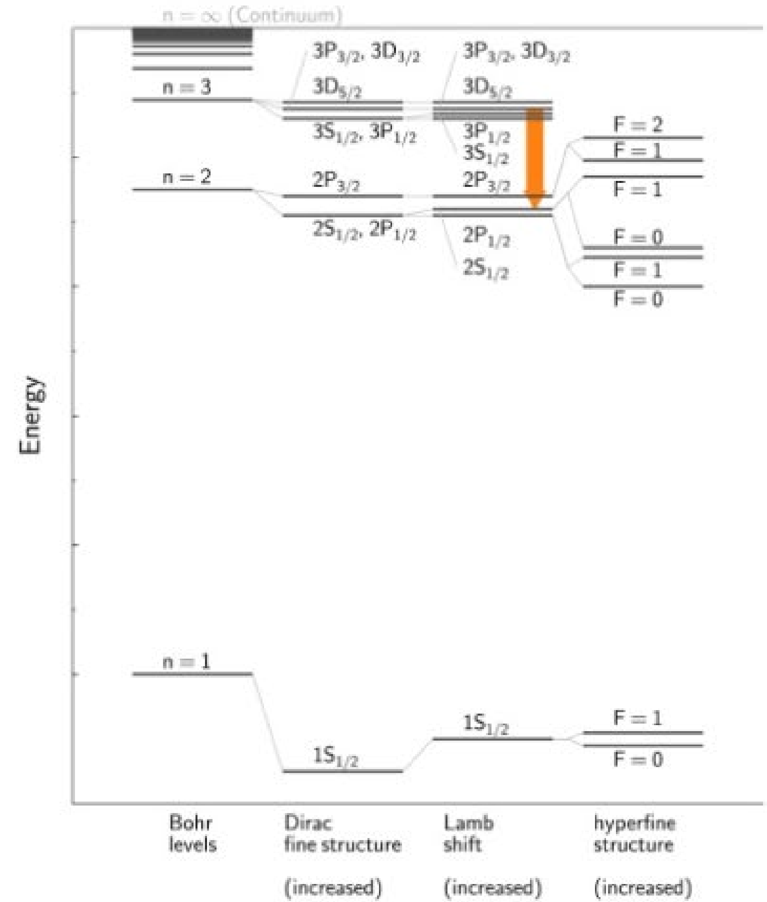
Are there any interactions in the hydrogen atom beyond proton-electron Coulomb attraction (with no external field) and how would we know?

Yes !

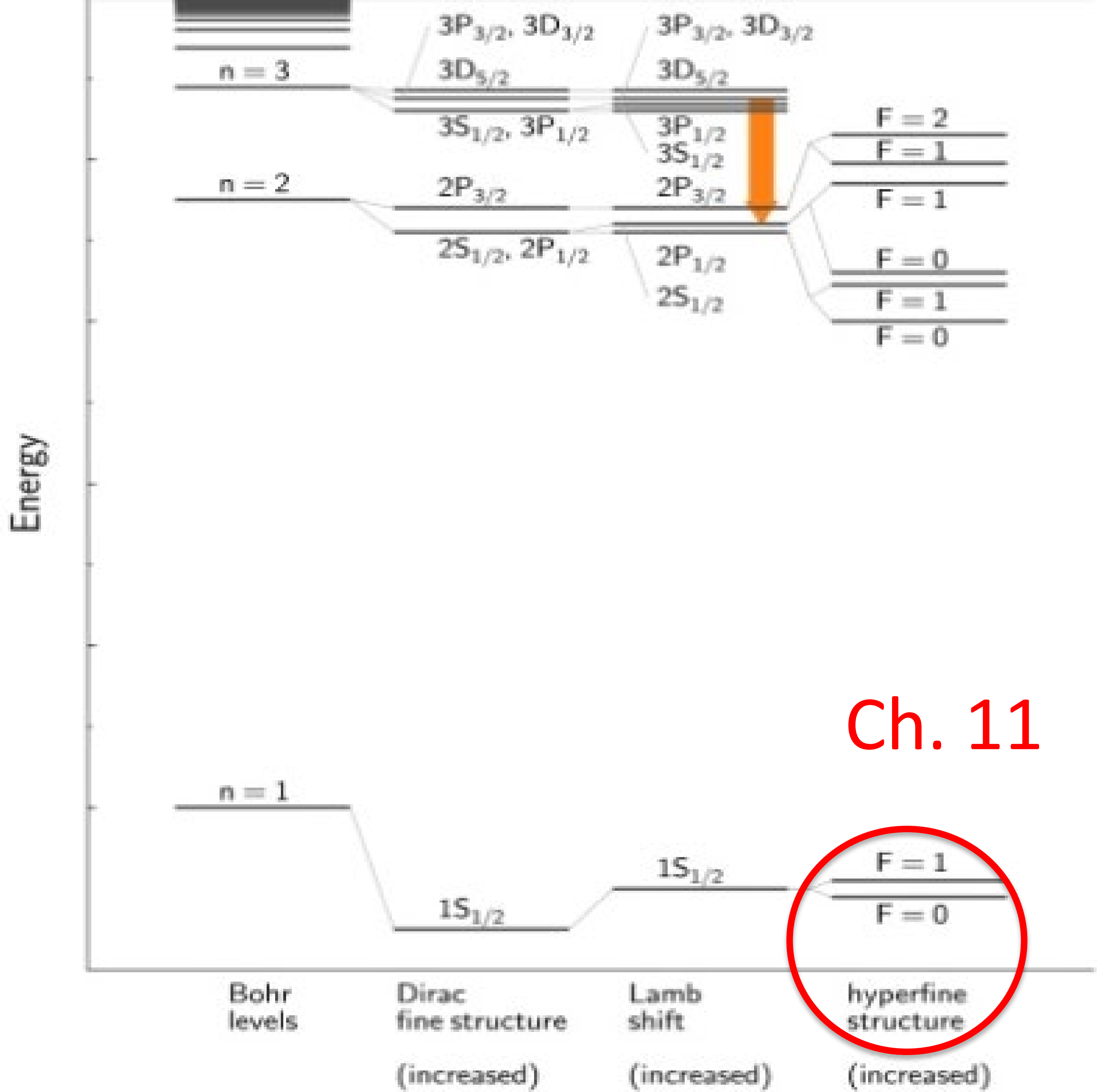
High-resolution spectroscopy ! (Hansch, Nobel Prize 2005)



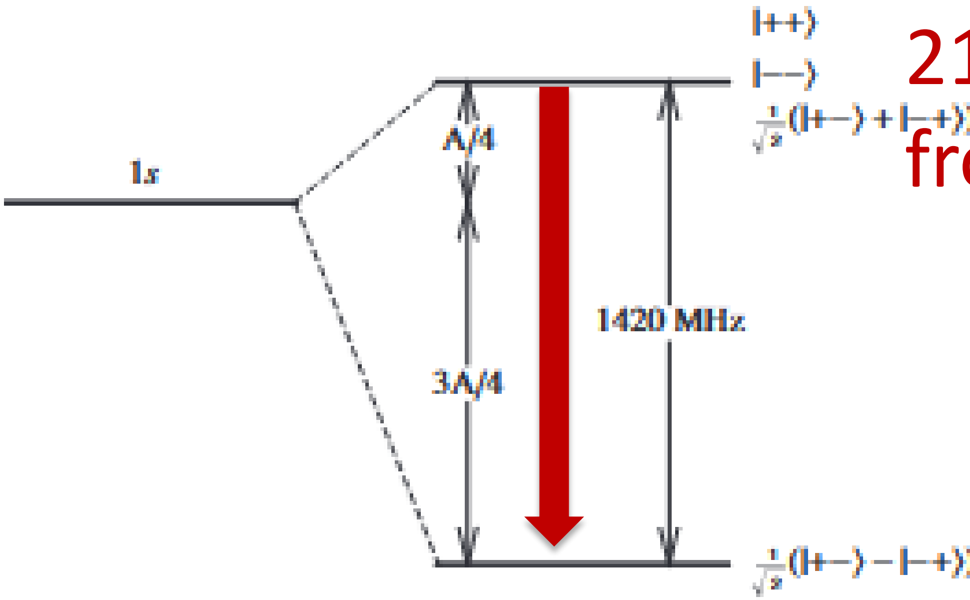




High-resolution spectroscopy can detect small changes in energy levels due to subtle effects

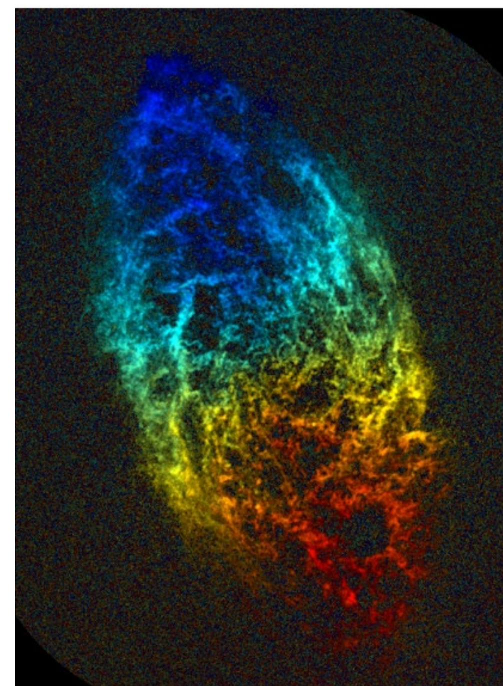

3P3/2, 3D3/2
3D5/2
n = 3
3S1/2, 3P1/2
3P1/2
3S1/2
F = 2
F = 1
n = 2
2P3/2
F = 1
F = 0
2S1/2, 2P1/2
2P1/2
2S1/2
F = 1
F = 0
Energy
Ch. 11
n = 1
1S1/2
F = 1
F = 0
Bohr levels
Dirac fine structure (increased)
Lamb shift (increased)
hyperfine structure (increased)

Hyperfine splitting of 1s state (ground state) of hydrogen atom



Radioastronomy:
21 cm line
from interstellar hydrogen

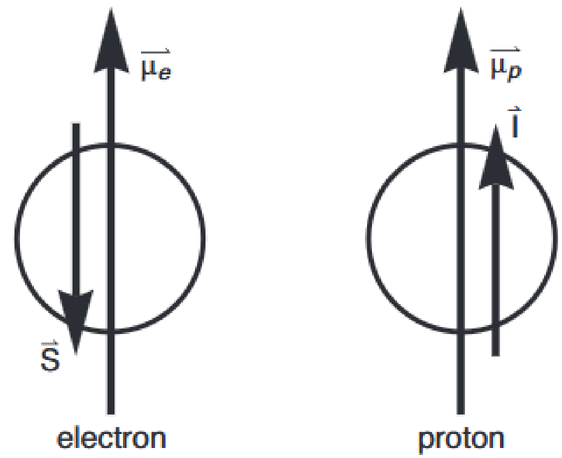
The hydrogen radial velocity field of the nearest spiral galaxy M33 observed at 21-cm wavelength. Colors corresponding to Doppler redshifts and blueshifts relative to the center of mass. Brightness in this image is proportional to hydrogen density.



https://public.nrao.edu/

Hyperfine interaction

1. The spins of the electron and proton introduce additional energy beyond the Coulomb interaction:



2. In symbols:

$$H'_{hf} = \frac{\mu_0}{4\pi}\frac{g_e\mu_B g_p\mu_N}{\hbar^2}\left[\frac{1}{r^3}\mathbf{I}\bullet\mathbf{L} - \frac{1}{r^3}\mathbf{S}\bullet\mathbf{I} + \frac{3}{r^5}(\mathbf{S}\bullet\mathbf{r})(\mathbf{I}\bullet\mathbf{r}) + \frac{8\pi}{3}\mathbf{S}\bullet\mathbf{I}\delta(\mathbf{r})\right]$$

$$H'_{hf} = \frac{\mu_0}{4\pi}\frac{g_e\mu_B g_p\mu_N}{\hbar^2}\frac{8\pi}{3}\mathbf{S}\bullet\mathbf{I}\delta(\mathbf{r}) = \frac{A}{\hbar^2}\mathbf{S}\bullet\mathbf{I}$$