PH 451/551
QM

Lecture #12
Stark effect (ctd.)

Recall: H-atom in $\mathcal{E}$-field $\parallel Oz$

4-fold deg.

$n=2 \Rightarrow\ ?$ $\quad H' = e\mathcal{E}z$, $\quad z = r\cos\theta$

$E_2^{(0)} = -\frac{E_I}{4}$ $\quad \mathcal{E}$

$$\hat{H}' \doteq \begin{array}{c} \begin{array}{cccc} |200\rangle & |210\rangle & |211\rangle & |21-1\rangle \end{array} \\ \begin{pmatrix} 0 & ? & & \\ ? & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \end{array} \begin{array}{l} \langle 200| \\ \langle 210| \\ \langle 211| \\ \langle 21-1| \end{array}$$

Need $\langle 200^{(0)}| e\mathcal{E}z |210^{(0)}\rangle = e\mathcal{E}\cdot$

$$\cdot \int_0^\infty R_{20}^* \, r \, R_{21}\, r^2 dr \int_0^\pi\int_0^{2\pi} \underbrace{Y_{00}^*}_{\frac{1}{\sqrt{4\pi}}} \cos\theta \, Y_{10}\, \sin\theta d\theta d\varphi =$$

$$= \frac{e\mathcal{E}}{(2a_0)^3}\frac{1}{4\pi a_0} 2\pi \int_0^\infty \left(2-\frac{r}{a_0}\right) r^4 e^{-\frac{r}{a_0}} dr \,\cdot$$

$$\cdot \underbrace{\int_0^\pi \cos^2\theta \sin\theta d\theta}_{=\frac{2}{3}} = e\mathcal{E}a_0\left(\frac{4!}{12} - \frac{5!}{24}\right) = -3e\mathcal{E}a_0$$

use $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx = (n-1)!$

So, $\hat{H}' \doteq \begin{pmatrix} 0 & -3e\mathcal{E}a_0 & 0 & 0 \\ -3e\mathcal{E}a_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\Downarrow$

diagonalise!

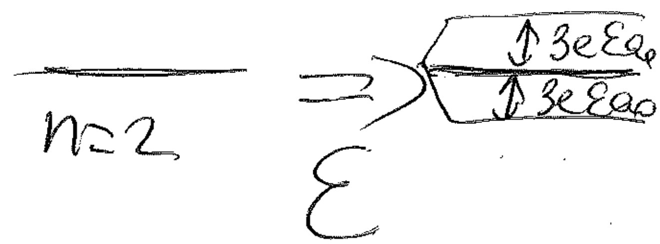
$$\det \begin{bmatrix} -\lambda & -3e\mathcal{E}a_0 & 0 & 0 \\ -3e\mathcal{E}a_0 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{bmatrix} = 0 \Rightarrow$$

$$\lambda^2\left(\lambda^2 - (3e\mathcal{E}a_0)^2\right) = 0 \rightarrow \lambda_{1,2} = 0$$

$$\lambda_{3,4} = \pm 3e\mathcal{E}a_0$$

$E^{(1)}_{n=2\,(1,2)} = 0$ ← double-degenerate

$E^{(1)}_{n=2\,(3,4)} = \pm 3e\mathcal{E}a_0$



Eigenstates: $\lambda = 0 \Rightarrow |2\,1\,\pm 1\rangle$ ← double-degenerate

$\lambda = \pm 3e\mathcal{E}a_0 \Rightarrow \frac{1}{\sqrt{2}}\left(|200\rangle \mp |210\rangle\right)$

The degeneracy is partially removed!