Review of paradigms QM

Read McIntyre Ch. 1, 2, 3.1, 5.1-5.7, 6.1-6.5, 7, 8

Systems you have studied:

- Spin-1/2 and spin-1
- Infinite square well potential
- Finite square well potential
- H-atom
- Angular momentum (rigid rotor)
- Free particle

Important concepts to know

- Quantum states
(ket, vector, wave function representations)
- General state as a superposition of eigenstates; projections, probability, expectation value
- Time evolution
- Important operators – Hamiltonian, position, momentum, spin,…
(representations – e.g. abstract, matrix, position)
- Energy spectrum, quantum numbers
- Other important concepts
(degeneracy, commutation, uncertainty,…)

QM Postulates

① The state of a quantum mechanical system, including all the information you can know about it, is represented mathematically by a normalized ket $|\psi\rangle$.

② A physical observable is represented mathematically by a linear, Hermitian operator *A* that acts on kets.

③ The only possible result of a measurement of an observable is one of the (real) eigenvalues a_n of the corresponding operator *A*.

④ The probability of obtaining the eigenvalue a_n in a measurement of the observable *A* on the system in the state is

$$\wp_{a_n} = |\langle\varphi_n|\psi\rangle|^2$$

where $|\varphi_n\rangle$ is the normalized eigenvector of *A* corresponding to the eigenvalue a_n.

⑤ After a measurement of *A* that yields the result a_n, the quantum system is in a new state that is the normalized projection of the original system ket onto the ket (or kets) corresponding to the result of the measurement:

$$|\psi'\rangle = \frac{P_n|\psi\rangle}{\sqrt{\langle\psi|P_n|\psi\rangle}}$$

⑥ The time evolution of a quantum system is determined by the Hamiltonian or total energy operator $H(t)$ through the Schrödinger equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

Superpositions

State superposition

$$\left|\Phi\right\rangle = \sum_n c_n \left|\varphi_n\right\rangle$$

Projections:

$$c_n = \left\langle \varphi_n \middle| \Phi \right\rangle$$

$$\left\langle \varphi_n \middle| \Phi \right\rangle = \left\langle \varphi_n \right| \sum_{m=1}^{\infty} c_m \left|\varphi_m\right\rangle = \sum_{n=1}^{\infty} c_m \left\langle \varphi_n \middle| \varphi_m \right\rangle$$

$$= \sum_{m=1}^{\infty} c_m \delta_{n,m} = c_n$$

Coefficients:

- Math: projection of the general state onto the eigenstate
- Physical meaning: probability amplitude

Time evolution

Schrödinger Wave Equation: $\hat{H}\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t}$

You learned in *Spins* the solution to the (time-dependent) SE is, in terms of the eigenstates of the Hamiltonian:

$$|\psi(t)\rangle = \sum_n |\varphi_n\rangle e^{-iE_n t/\hbar} \quad \text{where} \quad \hat{H}|\varphi_n\rangle = E_n|\varphi_n\rangle$$

$$\psi(x,t) = \sum_n \varphi_n(x) e^{-iE_n t/\hbar} \quad \text{where} \quad \hat{H}\varphi_n(x) = E_n\varphi_n(x)$$