

Homework #2

(due Wednesday, January 21, 2026)

1. (10 pts) McIntyre 9.9
2. (10 pts) McIntyre 9.14a (**not 9.14b !**)
3. (10 pts) Consider 1D harmonic oscillator. By setting up the energy eigenvalue equation in the momentum space and direct comparison with that in the position space, infer the momentum space eigenfunctions $\Phi(p)$ (you don't have to solve anything here, but the mapping from the position to the momentum space should be clear !).
4. (10 pts) Create a table for the H.O. (p. 2) similar to the that in HW #1 in which you represent the important quantities for the HO in ket (abstract), wave function, graphical and matrix notation.
5. Reading: Ch. 9.1-9.7 of McIntyre.

Harmonic oscillator	Ket Representation	Wave Function Representation	Matrix Representation
Hamiltonian			
Eigenvalues of Hamiltonian			
Normalized eigenstates of Hamiltonian			
Matrix element/matrix of position operator	$X_{nm} = \langle n X m\rangle =$	$X_{nm} = (\text{integral}) =$	$X := (\text{matrix})$
Matrix element/matrix of momentum operator	$P_{nm} = \langle n P m\rangle =$	$P_{nm} = (\text{integral}) =$	$P := (\text{matrix})$

The following questions are for practice (not for submission):

1. **McIntyre 9.13**

2. **Practice with commutators:**

(a) Show that the following commutator relations are true.

$$[\hat{A}, (\hat{B} + \hat{C})] = [\hat{A}, \hat{B}] + [\hat{A}, \hat{C}]$$

$$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}]$$

(b) Show that

$$[\hat{x}, \hat{p}] = i\hbar$$

Hint: operate on a general wave function and use the position representation of the operators.

3. **More practice with commutators**

$$\begin{aligned} [\hat{A}, \hat{A}^n] &= 0 & [\hat{A}, \hat{B}] &= -[\hat{B}, \hat{A}] \\ [\hat{A}, c\hat{B}] &= c[\hat{A}, \hat{B}] & [\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] &= 0 \end{aligned}$$

4. **The 2-D Quantum HO**

The Hamiltonian for a 2-dimensional isotropic harmonic oscillator is $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2)$.

- (a) Find the energy eigenvalues for the system.
- (b) What is the degeneracy?
- (c) What is the ground state eigenfunction?