MTH 655/659 WINTER 2008, Assignment 2

Show all the work and the code you implemented, if relevant.

1. Use ACF code to solve the problem as in II.7.10. Show convergence in L^2 and H^1 norms. (You can use either triangular grid, or rectangular, or both for **extra credit**).

For quadrature you can use the code tri_quadcofs.m.

Extra: estimate the order in L^∞ norm (see eq.II.7.8 in the book.).

2. Consider (in 2D, for triangles) the affine transformation F from the reference element $\hat{T}$ to a "real element" T with nodes (x_1, y_1).... Compute the local stiffness matrix a_T using the map F and directly by finding the basis functions and their derivatives (results should be, of course, the same).

Relate your computation to the calculations in ACF code for triangles.

3. (Do this problem analytically or numerically, or in both ways, which will be most fun, but is not required)

Consider the problem $-\nabla \cdot (\mathbf{K}\nabla u) = f$ on $\Omega = (-1,1) \times (-1,1)$ with some given Dirichlet boundary conditions, and where $\mathbf{K} = \mathbf{K}(x,y)$ is a given diagonal tensor. Assume the exact solution is given by $u(x,y) = xt(y)$ (see choices for $t(\cdot)$ below). In your solution pay attention to the question: for which $t, \mathbf{K}$ the problem and the solution are regular enough so that all desirable estimates hold ? Should Dirac δ appear as rhs and you need help with implementing it, ask me.

a) Describe stiffness matrix calculations/implement the code/ that will modify ACF code so you can include $\mathbf{K}$ (either triangles, or rectangles, or both).

b) Discuss what to expect/actually test the convergence/ for the following choices:

(i) $t(y) = y$, $\mathbf{K} = 2\mathbf{I}$

(ii) $t(y) = y$, $\mathbf{K} = diag(1,2)$

(iii) $t(y) = \begin{cases} y, & y < 0 \\ 10y & y \geq 0 \end{cases}$, $\mathbf{K} = \mathbf{I}$

(iv) $t(y) = \begin{cases} y, & y < 0 \\ 10y & y \geq 0 \end{cases}$ $\mathbf{K}(x,y) = \begin{cases} 10\mathbf{I}, & y < 0 \\ \mathbf{I} & y \geq 0 \end{cases}$

Extra: enjoy and if you wish, turn in for extra credit.

E1. Plot images of a reference triangle/square $\hat{T}$ under a nontrivial affine map and isoparametric map (see III. $2) for details.

E2: Compute an $L^2(0,1)$ projection $P_h g$ onto V_h of $g(x) = 1, x, x^2$ defined by $(P_h g, v_h) = (g, v_h), \forall v_h \in V_h$, where V_h is the space of piecewise linears. For $h = 1, 1/2$ you can do it analytically; implement this set-up so you can do it numerically for any h. Calculate $\| g - P_h g \|_{L^2(0,1)}$. Use $h = 1, 1/2, 1/4...$, and determine the order of convergence.

E3: find the piecewise Hermite interpolating polynomial I_h^H for $g(x) = sin(\pi x)$ on a grid with parameter h covering $(0,1)$ and compare it to the Lagrange interpolating polynomial using plots, estimates of order of convergence in various norms.

E4: Do Pmb2 on rectangles/parallelograms, with special attention paid to how ACF code treats these.