MTH 623/Peszynska/Spring 2012
Worksheet 7.
Please show all your work. Use proper mathematical notation.

Consider solution $u(x,t)$ to

$$u_{tt} = c^2 u_{xx}$$

with initial data given by $u(x,0) = u_0(x), u_t(x,0) = u_1(x)$.

Fill in the details to derive d'Alembert's solution to this equation by using an appropriate system of hyperbolic conservation laws $\vec{q}_t + A\vec{q}_x = 0$.

1. Substitute $q_1 = u_x, q_2 = u_t$ and find A.

2. Find the eigenvalues and eigenvectors of A. Is the system (strictly) hyperbolic ?

3. Transform the initial data to that for $\vec{q}$ and solve the system for $\vec{q}$.

4. Transform back to derive u (it should coincide with d'Alembert's formula).

5. Repeat the steps 1-4 for $u_{tt} = (\sigma(u_x))_x$ and make assumptions on σ that render the problem hyperbolic. (If you use $p \equiv -\sigma$, the system is called the p-system and it represents isentropic gas dynamics in Lagrangian coordinates).