

MTH 623/Peszynska/Spring 2012

Worksheet 2.

Please show all your work. Use proper mathematical notation.

Consider an IVP problem for

$$u_t + vu_x = 0,$$

where  $v > 0$  is constant, with a smooth initial data  $u_0(x)$ . Show that the solution is the limit of viscosity solutions.

Construct the viscosity solution to this problem i.e.  $u^\epsilon$  which satisfies

$$u_t^\epsilon + vu_x^\epsilon = \epsilon u_{xx}^\epsilon$$

and pass to the limit using steps below.

1. Use the change of variable  $w^\epsilon(x, t) = u^\epsilon(x + vt, t)$ . and show that  $w$  satisfies the heat equation.
2. Write explicit formula for  $w^\epsilon(x, t)$  using the fundamental solution to the heat equation.
3. Substitute back to obtain  $u^\epsilon$  from  $w^\epsilon$ .
4. Take the limit as  $\epsilon \rightarrow 0$ . Show it is equal to  $u_0(x - vt)$ . Justify the elements of the limiting process.

What happens if  $u_0$  is piecewise constant instead of smooth ?