MTH 453-553 W2013, Assignment 2

Students registered for 453 solve Problem 1a-c and 3.
Students registered for 553 solve Problem 1a-c, 2b, 3.
More problems for extra credit !

1. Consider solving Poisson's equation $- \triangle u = f$ on $\Omega = (0,1) \times (0,1)$ with the following mixed boundary conditions: Dirichlet at the left, right, and bottom part of the boundary, and Neumman conditions on the top part.

   (a) Write out carefully the algorithm to implement these conditions with a second order accurate method (ghost cells). Write it for $h_x \neq h_y$.

   (b) Implement and test convergence using $u(x,y) = sin(\pi x) * sin(\frac{7\pi y}{2})$. Do this with $h_x = h_y$.

   (c) Consider $h_x \neq h_y$, and repeat b). Compare the error and convergence for $h_x = h_y$ first, next $h_x = 5h_y$, and then $5h_x = h_y$.

   Make sure you plot the solution and compare the analytical to the numerical solutions. [I do not need to see all your plots, but do so to make sure your algorithm works right].

   (d)) **Extra:** implement the first order method for Neumann condition and show it is at most first order.

2. **Extra credit exercises** Implement solving the following elliptic equations. For each problem, modify the code appropriately and test convergence, if relevant.

   (a) Do problem 1) on $\Omega = (0,1) \times (0,2)$ and then $\Omega = (-1,1) \times (0, 2/7)$.

   (b) Solve $- \triangle u + u = f$,

   (c) Solve $- \triangle u + a(x,y)u = f$, where $a(x,y) = x + y$.

   (d) Solve $-\nabla \cdot (k(x,y)\nabla u) = f$, where $k(x,y) = x + y$.

   When testing convergence in b)-d), use the function $u(x,y) = sin(pi * x) * sin(10 * pi * y) + x + y$. For each problem, find the appropriate right hand side.

3. Consider the following heat conduction problem for which we do not know the analytical solution. Consider homogeneous Dirichlet boundary conditions for the temperature on the sides of a rectangular pool, and a right hand side function f which models a localized heat source in the middle of the pool, so that $f(x,y) = 100$, if $(x - 0.5)^2 + (y - 0.5)^2 \leq .1$, and 0 otherwise. Plot the solution and determine what h_x, h_y you should use to

get a reasonably looking solution. Prepare a table in which you record how the maximum value of temperature changes with h_x. How much should it change ? Do you see a plateau ?
Extra: play with other boundary conditions.