MTH 453-553 W2013, Assignment 1

Students registered for 453 solve Problem 1 and one other problem (or all for extra credit).

Students registered for 553 solve all problems.

1. Follow instructions for Exercise 3.1ac from the book to solve $-\triangle u = f$ on $[0,1] \times [0,1]$ so that $u(x,y) = exp(x + y/10)$ is the true solution.
   Make it clear what boundary conditions and what source (right hand side) function f you are using.
   In a), show that the algorithm is second order accurate (use `error_log.m` and `error_table.m` to see how to show the results.).
   In c), propose the best choice of $\triangle x, \triangle y$ for the problem so that the total error is smaller than $\tau = 1e-6$, but the number of unknowns is as small as possible.

2. Solve 3.2.

3. Consider a diagonal positive definite matrix $K \in \mathbb{R}^{2\times 2}$ and consider solving $-\nabla \cdot (K\nabla u) = f$.
   Propose a FD scheme and show its consistency.
   Modify `poisson.m` to solve this problem on $[0,1] \times [0,1]$. Use $u(x,y) = \sin(\pi x)\sin(\pi y)$ as a true solution, and $K_{11} = 1, K_{22} = 10$. Derive the boundary conditions and the source terms.
   Test convergence and propose $\triangle x, \triangle y$ so that the error is less than $\tau = 1e-6$, with the smallest number of unknowns possible.