MTH 452-552/Winter 2013, Assignment 6

You can use the code `fd1d.m` as discussed in class, or implement your own. Below I am assuming you are using `fd1d.m`.

1. (452-552) Implement the numerical solution for the two-point BVP

$$-u'' + u = f,\ x \in (0,1),\ u(0) = 0,\ u'(1) = 0 \tag{1}$$

(You will have to account for a different form of the differential equation than that implemented in `fd1d.m` as well as for a new type of boundary condition).
Assume the exact solution is known $u(x) = sin(11\frac{\pi}{2}x)$. (You will want to check that it satisfies the boundary conditions as well as what $f(x)$ it corresponds to).
(i) Decide (by had calculation) what h you must use in order for the truncation error to be below 0.1, and call it h_{mine}.
(ii) Use a sequence of grids with $h_0 = 2h_{mine}, h_1 = h_{mine}, h_2 = h_{mine}/2$ in the experiments below.
(iii) Run the code and show convergence in both the $\infty-$grid norm as well as in the 2-grid norm (you must implement the latter).
The convergence should be shown either in a table or using a `loglog` plot, or both (as discussed in class). The table displaying the order of convergence should be

h	$E_\infty(h)$	ratio	$E_2(h)$	ratio
h_0	...			
h_1	...			
h_2	...			

2. (452) For the problem

$$-u'' = f,\ x \in (0,1),\ u(0) = 0,\ u(1) = 0 \tag{2}$$

with $f(x) = x$ find the analytical solution by hand calculation. What do you expect the error to be ? Confirm with the numerical solution using at least three grid sizes.

3. (552) Same as Pbm 2 above but find f and boundary conditions so that

$$u(x) = |x - 0.5|(x - 0.5). \tag{3}$$

Pay attention to the smoothness of the solutions. What does the theory say? What do you get in practice? Experiment with an odd and even number of subintervals when implementing the solution and checking the error.