MTH 452-552/Winter 2013, Assignment 5

Solve two problems or more for extra credit. (452) students can solve 3 instead of (2).

1. (452-552) (i) For the one-step γ-method from midterm exam, calculate the growth factor $R(z)$. Recall that each of the optimal $\gamma_{1,2} = 1 \pm \sqrt{2}/2$ gives formally the second order accuracy.

   (ii) Use both the optimal $\gamma_{1,2}$ and the non-optimal $\gamma_3 = 0.5$ in the following.
   Plot the stability region for the method (i.e., plot the curve $|R(z)| = 1$ and verify which region is "outside"). Plot order stars (i.e., the curves $|e^{-z}R(z)| = 1$).

   Use the plots to confirm the convergence order, and to discuss stability of the method: what is the region of absolute stability, is the method A-stable, is it L-stable. Which γ would you choose to have a 2-order convergent method which can work with the largest time step possible ?

2. (452) In computational part of this problem use initial data $u(0) = 1, u'(0) = 0$.

   (i) For the harmonic oscillator problem $u'' + \omega u = 0$, with $\omega > 0$, written as a linear system of ODEs, find the eigenvalues λ. Is it possible to choose the time step k for FE method for $\omega = 100$ to prevent the solutions from increasing ?

   (ii) Now change the system to that with damping where $u'' + u' + \omega u = 0$, and answer the same questions as in (ii). Provide plots of solution computed with FE and a "good" h, and one with a "bad" h, if they can be found. [You can use `circle_demo.m` in computational experiments to illustrate what you are finding by hand calculation. Of course the analytical solution for your problem is different from that coded.]

   (iii) Consider now the non-linearized harmonic oscillator $v'' + \omega sin(v) = 0$. Implement a solution for this problem using FE. Plot the difference between v and u from (i) when $\omega = 100$. Is the linearized problem a good approximation to the original one ?

   Extra: implement a predictor-corrector $P(EC)^{20}$ for the non-linearized problem (i.e., use a fixed-point iteration. Show results, compare to other solutions you found.)

3. (552) Consider the Lorentz system as discussed in class with initial data $\mathbf{v} = [0, 2, 10]^T$. You can use the code `lorentz.m` as a starting point: note

that the code computes a (FE) solution and a predictor-corrector solution of first order.

(i) Linearize the Lorentz system about the initial data $\mathbf{v}$.

(ii) Calculate the eigenvalues of the Jacobian for the linearized system. [You can do it by hand or using MATLAB]. What should be the time step for the linearized system ? What about with initial data $\mathbf{w} = [0, 0, 0]^T$?

(iii) Implement the solution of the linearized system and compare the solutions to those of the original system up to roughly $T = 0.35$. Discuss what happens after that until $T = .8$. [Hint: a good time step to start your experiments with is $k = .01$, but experiment with time step found in (ii)]. Comment on what should happen for initial data equal $\mathbf{w}$. What **does** happen ?

(iv) **Extra:** implement checking the eigenvalues of the Jacobian of the original system as the solutions evolve in time. At some t the eigenvalues change dramatically qualitatively, and this is accompanied by the behavior of the solution. Discuss appropriate time step choices.

Implement the true BE method, or any higher order accurate L-stable method such as the γ method for pbm (1), or TR-BDF2. You should code the nonlinear solver carefully: either with Newton's method, or fixed point with error control.