

MTH 452-552/Winter 2008, Assignment 3

1. Consider the θ -method defined as

$$U^{n+1} = U^n + h [\theta f(U^n) + (1 - \theta)f(U^{n+1})]$$

for an autonomous ODE, where $0 \leq \theta \leq 1$ is a parameter.

Verify that the local truncation error of the method is at least first-order.

For what value of θ is the method second-order ?

Extend the method to the non-autonomous case; you can use Butcher tableaus to guide you.

(552 only) Analyze directly the convergence for the non-autonomous case using Taylor's expansions.

2. (MATLAB) Implement the θ -method from Pbm (1) for $\theta > 0$.
(452 and 552) Confirm the order of convergence you discovered in Pbm (1) for the linear ODE from HW2, Pbm (2)
(552) Implement and confirm the order for the problem $u' = f(u) = 2u^2, u(0) = 1/2$. What issues do you anticipate ?

Extra: What do you expect should happen if $f(u) = \begin{cases} 2u^2, & u \leq 1 \\ 1/u^2, & u > 1 \end{cases}$.

Confirm or refute your expectations.

3. (**Extra**) (MATLAB) Implement a-posteriori error analysis using Richardson's extrapolation and test it for the ODEs from Pbm (2).