MTH 351, Assignment # 4

1. In MATLAB, use numerical integration to approximate $\int_0^\pi \sin x dx$ when m subintervals are used. Let $m = 5, 10, 20$. Use trapezoidal and Simpson's method (provided in MATLAB or written by you) and write a routine to implement the left, midpoint and right rectangle quadrature rule. Compute the exact error, the theoretical error estimate, the asymptotic error estimates, as well as the error estimate using Richardson's extrapolation (as shown in class). Comment on the quality of the approximation depending on m and on whether the error estimates are close to the actual error.

2. Do (1) for the function $f(x) = \sqrt{|x - 0.5|}$ on the interval $(0, 1)$. Make sure to consider the singular behavior of this function at $x = 0.5$.

3. a) Use the forward, the central difference formula, and the one-sided second order difference formula which we derived in class using Richardson's extrapolation, to approximate $f'(1)$ for $f(x) = \sin(x)$, with h ranging from $1E-1$ down to $1E-4$. Discuss behavior of the error which should converge to 0 as $O(h), O(h^2), O(h^2)$, respectively. Confirm this rate.
b) Next consider $h < 1E-4$. Recall that due to numerical instabilities the error stops converging when $h < h_{crit}$. Consider h ranging from $1E-4$ down to $1E-14$ to discover what h_{crit} is for each of the methods.
(You may want to use logarithmic scale plotting routines in MATLAB loglog, semilogx, semilogy).