MTH 351, Assignment # 3

In this assignment you are asked to use polynomials to approximate a function and to (ultimately) find its (numerical) integral(s). **Hint:** you may find the following MATLAB and Atkinson/Han routines useful: `polyfit` with `polyval`, `interp1, spline, trapz, quad*`. Remember to check what these different routines do.

1. Let $f(x) = sin(x)$. In MATLAB, interpolate f on the interval $[0, \pi]$ using $n = 3$ equally spaced nodes and a Lagrange polynomial $p_2(x)$ (use polyfit). Plot $f(x)$ and the polynomial and verify how the error that you can read from the plot agrees with the theoretical estimate that you can derive. Repeat with a (piecewise) linear $s_1(x)$ (use interp1), and a cubic spline $s_3(x)$ (use spline) of your choice. Comment on the performance of these interpolating polynomials.

2. (Runge phenomenon) a) Find a Lagrange interpolating polynomial for $g_\alpha(x) = \frac{1}{1+25x^2}$ using $n = 3, 7, 11$ equally spaced nodes $x_0, x_1, \ldots x_{n-1}$ in the interval $[-1, 1]$ (use polyfit in MATLAB). Plot the polynomials and the function. Is the fit good ? b) To improve the results for $n = 7$, use instead of equally spaced nodes $x_0, x_1, \ldots x_{n-1}$ the Chebyshev nodes $z_0, z_1, \ldots z_{n-1}$ that is, the roots of Chebyshev polynomial of degree n which are given by the formula: $z_i = \cos(\frac{(2i+1)\pi}{2n}), i = 0, 1, 2 \ldots n - 1$. Discuss this improvement.