Modeling and Problem Solving: Curriculum and Program Development

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Introduction

Mathematical modeling can be defined, arguably, as "word-problem solving." In this chapter, we discuss program and course development for advanced mathematical modeling, i.e., using mathematics beyond calculus at the undergraduate and graduate levels. Doing so can either be tied to coursework at the undergraduate level or else extended to research-grade applied mathematics.

We focus primarily on the elements of curriculum and program development that can promote the students' interest and growing experience with mathematical modeling. As we argue below, mathematical modeling has "no boundaries," and evolves in time, thus unlike most other subjects, one can never declare it fully "completed." However, "boundary conditions" constrain inclusion of interdisciplinary mathematical modeling in curriculum and programs and require substantial investments of time from instructors and from students.

The general public dreads word problems, starting in elementary school. Two trains, one leaving Chicago at a speed of 70 mph, and another leaving Milwaukee at a speed 20% faster, are the subject of many popular jokes, a headache for parents, and a menace on standardized tests. Ironically, including word problems is intended both to humanize the mathematics and to motivate students to work hard at the math itself.

There are countless textbooks and Websites that aim to teach "word-problem solving" by teaching their *language*. Literally, these sources translate English to mathematics, and provide desperate students with simple suggestions, such as "per" means "division," and "increased by" means you need to use addition. While such sources are certainly helpful, their premise is that you can solve every word problem by a template combining the "translation" with arithmetic and some interpretation/validation. The advocates of such template training argue that practice makes master. However, critics point out that drills in word problem solving inhibit critical thinking, and that they actually de-motivate the more curious and ambitious students from the subject.

At the same time, various professionals argue [Bressoud et al. 2014; SIAM 1996, 2011, 2012] that experience in mathematical modeling and interdisciplinary problem solving is important (at least) for

(i) career and workforce preparation, and

(ii) entering graduate school in applied sciences and engineering.

More generally, faculty in mathematics departments use mathematical modeling for developing

(iii) advanced problem solving, and

(iv) to motivate and attract students who are applications-oriented.

There is danger in

- interpreting (i-ii) as a parallel to *tracking*, which is unfortunately still present in K-12 education; and furthermore,
- considering that (iii-iv) provide somehow an "easier" trajectory to a mathematics degree.

To refute such opinions, we should continue to build excellent programs that foster interdisciplinary mathematical modeling.

It seems that all (the critics, the advocates, and those already involved in mathematical modeling), would agree that to solve problems, it is mandatory to know the basic mathematics to which a given problem can be translated. Most would also agree that no one idea fits all students and programs.

In this chapter, we discuss how to incorporate mathematical modeling in curricula and program development in order to support (i-iv) without watering down its mathematical content. The ideas proposed apply to mathematics beyond calculus, mostly at undergraduate level. We first identify components of the "basic math" at the advanced undergraduate level, and propose extensions that add realism and excitement to examples and projects, as well as expose students to the validation/verification doldrums. Then we make a call for continuing construction of a database of projects and case studies, and propose ideas on research and capstone projects. We end with some notes on computational skills. Finally, graduate students with no prior experience with modeling may be the most difficult clientele of modeling courses, and we address this special topic at the end.

The ideas expressed below reflect the author's experience as an applied and computational mathematician and educator, and are expressed with a premise that there is an infinite amount of energy, time, and resources that one can spend on program building. This is of course not a reality. Programs and department chairs change, faculty are evaluated for tenure and promotion, and availability of various reward systems including research and education funding as well as professional awards change the priorities of any individual, while universities and college undergo various deep transformations almost independently of what the bottom-up efforts of their faculty create. These notes are therefore intended to spur interest, generate discussion, and lead to an improved state of 21st century of mathematical modeling.

Mathematical Modeling in Undergraduate Programs

Consider a typical mathematics or applied science or engineering major in their sophomore to junior years. They will have likely finished calculus and differential equations, and they are close to having decided about their future coursework and specializations such as options for majors. Ahead of them are various mathematics courses to choose, many of them of applied nature, and many of them not. Not always can the students connect the coursework to their future, and/or to their (maybe still vague) career options. More often than not they have seen applied mathematical modeling appear in their coursework. However, since syllabi are overloaded, there may have not been enough current interest or time to develop models, so the students remember modeling as the dreaded "two trains" word problems.

Enter applied mathematics faculty who want students to be involved in mathematical modeling.

- First, because they want to attract to mathematics those who are yet undecided that mathematics can be useful.
- Second, they want to keep high achievers in the game, and show them that applied mathematics is a subject without boundaries that can present as many challenges as one wants.
- Third, they want to motivate those students who have not yet been willing to work hard on the mathematics itself. Bringing in mathematical solutions to real-life problems and incorporating current everyday technology appears exciting. The professors hear "I did not know you could use math for that!" and "I did not know math could be so cool!"

Students are thus encouraged to participate in MCM or other contests organized by COMAP and/or other competitions, seminars, etc., and to take modeling courses. Many students succeed, and for many these experiences change their study and career plans.

So next we have students who want courses in which they can remain equally engaged and stimulated. They want to model, model, model, and solve, solve, solve. In the best of circumstances, there is one course on modeling in the department. Typically, such a course is taught by an enthusiastic faculty member already involved in modeling in their research. The instructor may already have developed a set of projects themself and/or be using some textbooks on modeling. The curriculum and major options may have a track developed especially for applied mathematics with a significant modeling component, and there may be resources to support individual research or senior thesis projects and so on. But there is only so far that an individual or a group can go, and it is hard to develop stable long-term programs based on a small sample. Those who are not involved in modeling themselves need to see a sustained long-term high quality program and coursework with evidence of success.

There are many applied mathematics programs and majors in the United States alone, and many organizations promote applied mathematics and mathematical modeling [SIAM 2011]. If you are a student/or faculty member in such programs, then you are all set. If you are not, or you are thinking of revising or developing such programs, the notes below may be for you.

Modern Applied Mathematical Modeling Needs . . . Methods

Mathematical modeling is a subject without boundaries. It is the means by which mathematics becomes useful to virtually any subject, and modeling has been and continues to be a driving force for the development of mathematics itself. Applied mathematics has evolved over the last three decades from the classic engineering-driven studies of differential equations [Fowler 1997; Haberman 1977] to a field that is foundational to, e.g.,

- **[MMT]** (Modern Modeling Topics): image processing and deblurring, geodesy and GPS, seismology, web search engines, Netflix "recommender systems," analyses of biological, social, and urban networks, data mining, genomics, and many more topics.

The success of bioinformatics, and more broadly, of mathematical biology [Murray 2008], is parallel to this shift and expansion of applied mathematics into fields other than engineering and physical sciences. Computational mathematics has set the foundation for successful virtual laboratories, where virtual experiments are performed "in silicon" at multiple scales, to provide qualitative and quantitative information about processes that are otherwise unavailable or for which experiments are prohibitively expansive or destructive.

Because of the usefulness of applied mathematics, it is now taught in engineering and other science departments—and sadly, it is because of this usefulness that the methods of applied mathematics are sometimes taught or labeled as not rigorous enough or non-cool.

The traditional applications including mechanical vibrations, electric circuits, water tanks emptying, and logistics modeling are viewed by students (and consequently dreaded) as a subject very close to the "two trains" word problems, except that they are solved with calculus or differential equations. Even Fourier analysis is now tainted, as a subject that requires too much patience and lengthy calculations, because the payoff for these is not easily seen at the beginning and the applications take time to develop.

Thus, there is need to reinvent the base and to introduce a slew of new applications for which the students only need to have the same common prerequisite base. These new applications can be taught in dedicated "models and methods" courses where the students learn new mathematics while being motivated by the applications. Courses are needed to explain the mathematical structure behind the applications, and show how that structure can be analyzed. While the applications are specific, the methods can be universal.

Courses devoted only to modeling can be challenging to develop and maintain. Furthermore, it may be difficult to present them as "hard-core" formal mathematics courses. The principles of including enough new knowledge to make the course worthwhile, as well as starting from scratch in one or more fields of applications, are contradictory! For new mathematical knowledge, however, all you need to do is introduce new and challenging methods, i.e., enough mathematics itself. However, the richness of applications of mathematics, both classic and modern, precludes the ability to teach them "all." Picking a textbook and/or designing a course that covers some detailed modeling in an encyclopedic fashion such as Haberman [1977] or Fowler [1997] is not the best idea at the undergraduate level, since the course may miss a unifying theme and could fail to develop enough long-lasting mathematical substance. Developing a foundation of *applications background* (AB), as discussed below, is a different story.

Prerequisites for Modern Applied Mathematical Modeling Coursework

Recall the observation that a student can "translate" a word problem to its model form, e.g., "per" to "division," only if (s)he knows enough about division. Thus, some prerequisites are required so students can recognize how to frame the given applications and develop and work with various mathematical models.

Typical mathematics undergraduate curriculum beyond calculus includes ordinary differential equations (ODEs) and maybe systems of ODEs (SODEs), linear algebra (LA), and (some) probability and statistics (PS). These typically are based on a sound multivariable calculus course (MV).

Now consider coursework needed for modern applied mathematics and projects. Surprisingly, many can be developed from extensions of simple concepts learned in LA and ODEs. In addition, there is strong need to develop the basics of probability and statistics, as well as of discrete modeling including fundamental principles of counting, graph theory, and networks. Thus combination of ODEs, LA, and PS is also all that is needed, with small adjustments and extensions that open opportunities for mathematical modeling to be included in current coursework but also encourage future study in advanced courses.

Next consider the concepts of mathematical modeling throughout courses on ODEs, LA, and PS. In ideal world, one should include some modeling in every prerequisite course so as to promote experience and growing interest in these. ODEs and PS reach naturally to applications while LA tends to focus on the technical expertise. To promote mathematical modeling in these basic courses one can make adjustments. Since this will cost course time, it should be negotiated with other faculty and especially those who are coordinating syllabi. The time for modeling projects can be perhaps generated by covering some tedious topics with less intensity, and leaving to the students the task of practicing those on their own. However, make sure the modeling examples and projects are complex and intellectually rich. In other words, the modeling efforts should require substantial intellectual effort rather than just following of the templates; this can make up for the time expense and justify modifications in course content. To keep the examples modern and students engaged, include and use technology and real-life data.

Advanced Mathematical Modeling (AMM) Coursework

We propose that advanced coursework in MMT (Modern Mathematical Topics) should cover:

- **[AMM concepts]** nonlinearity, coupled systems and scales, uncertainty, and optimization.

Students should be attracted to the complexity and richness of a problem, learn to simplify and isolate a model, live with the consequences of imperfect models, and accept validation and verification as a necessary step.

How to Develop AMM

Some AMM concepts can be developed in the prerequisite courses. Some need time to develop and are perhaps best taught in a dedicated "models and methods' course, or an applied numerical analysis (NA) or applied analysis (AA) course.

First, most textbooks on differential equations present ample examples of applications of scalar ODEs. But it is only *coupled systems* that provide enough mystery to attract the interested students, while a traditional course may never get there. Most mathematics programs include a course on systems of ODEs, but few students experience large systems and or PDEs, which can be presented as natural limits of finite-dimensional systems.

Next, not every problem has a solution, and some solutions are not unique. A perfect introduction to this idea in LA coursework is coverage of overdetermined and underdetermined systems, and introduction of least-squares. Naturally from this follows a connection to the minimization techniques, and to the singular value decomposition. In another direction, it is fairly easy to introduce ill-conditioned linear problems, and tie this to applications in discrete inverse problems [Hansen 2010]. Least-squares can be weighted, and simple examples lead to the Kalman filter, which in turn requires understanding of uncertainty. Furthermore, optimization ties to approximation theory, and is most easily explained in inner product spaces.

Linearity is a blessing and a curse. Except for some topics in (scalar) differential equations, the techniques for solving linear problems can be learned in basic coursework, and they lead to nice and neat solutions. But the world is nonlinear and messy, and students are immediately baffled with handling even simple nonlinear systems of algebraic equations, for which there are no general "rules"of solution. Linearization comes as a blessing and a possibility, but trying to cast everything as a linear problem is not wise. Global Positioning System (GPS) systems applications can serve as an interesting contemporary application introducing students to nonlinearity. The 300-year-old Newton's method comes to the rescue, along with nonlinear least squares plus the Kalman filter [Strang 1986; Strang and Borre 2012].

Nonlinearity, inner product spaces, and approximations are efficiently taught in undergraduate numerical analysis courses (NA) [Atkinson and Han 2003]. They can also be taught in advanced calculus courses as applications of fixed-point methods, and multivariable advanced calculus. The basics of variational calculus can be explained using only a sound background in MV when scaled down to finite-dimensional problems and solved via LA.

Solving a (forward) problem, such as how quickly a tank fills up, is dealt with in ODE courses. In real life, though, all we do is *prioritize and optimize*, mostly heuristically. However, there is a rich body of modern as well as classic mathematics associated with inverse problems and optimization, and teaching fundamental concepts can benefit students both in their further education as well as in practical careers.

Structure of AMM courses

How to structure the mathematical modeling courses? Since there are few textbooks that fit the need, it may be difficult to fit the courses to the traditional "lecture and homework" style. What seems to work is "guided projects," where students delve deeply into an application area while practicing skills taught in a lecture.

What can students learn from a projects-based course? That modeling is hard, and that even if they did not succeed the first time, eventually they will. The instructors should avoid teaching templates beyond generalities how to get started, how to simplify the problems, and how not to. For example, assuming linearity and constant coefficients is a good first step in solving many practical problems—but it should not be the last !

We suggest that one should construct problems and projects in layers:

- First, lead to a model: explain and give real coefficients.
- Next, simplify it: non-dimensional analysis is useful but students tend to be rather weak in using the chain rule.
- Remove second-order effects temporarily.
- Solve the first-order idealized model, and analyze its sensitivity.
- If possible, bring in real-life data to evaluate the model's usefulness.

The challenges in AMM courses are

- to come up with a set of guided projects;
- to tie them to the lecture material accompanied by a textbook;
- to strike a balance between giving students a sense of success, and at the same time teach to pursue some form of validation and verification, and in particular how to recover from mistakes;
- the projects cannot be too long, and should take at most two class hours to develop. This translates to at most about 10 pages of background information needed by the instructor, to be distilled to a two-page handout, with references reaching to deeper treatment of the subject.

Class Projects

We believe guided projects are the core of AMM coursework. Fundamentals of methods such as optimization, discrete modeling, etc., can be used to solve fascinating and engaging modeling projects at the basic and advanced levels. It helps when they go beyond canonical examples from Haberman [1977] and Logan [2006] and relate to current technology such as in Strang and Borre [2012] and O'Leary [2009]. The projects need be based on real-life data and use physical units. The challenge is, however, that the projects require simplifications so that they can be tractable in finite time with a reasonably basic level of mathematics. Unfortunately the more simplified they are, the more distant they are to reality, and a student's interest dissipates.

There are not many resources that connect real life to mathematical models and methods at the level appropriate for advanced undergraduate coursework. The problems are either too easy or too hard, and one can only go so far before either the mathematics gets too hard or the units and technical specifics of the field get in the way. There is a limited number of problems and projects that have been described, and even these are not always adequate for a single class project, or for integration into consistent curricula. The References include a few online repositories, sources, and collections popularizing interdisciplinary mathematics [COMAP n.d.; SIAM 2011, 2014]. Among others, MATLAB based projects developed in Moler [2011] and O'Leary [2009] provide a very useful set of templates. SODEs examples in Strogatz [2001] are fantastic and motivating for coupled systems and nonlinearity.

A faculty member developing new problems and projects reaching out to interdisciplinary science and engineering must act as an interpreter. The barriers of language specific to a given engineering or applied science field, the units, the way of thinking, the research objectives, and so on, are tremendous. Faculty engaged in research in interdisciplinary mathematics typically make a decision early in their career about the area in which they specialize; with this comes a particular set of mathematical methods and a particular "translating" expertise.

However, when developing coursework and programs in mathematical modeling one needs to reach *across* various application domains. Faculty thus must make substantial time investment and go out of their own comfort zone. Furthermore, the dangers in confronting real-life data lurk right around the corner: the problem that we pick and *can* solve with the tools at the given level can be posed, e.g., at a wrong scale, or missing an important first order effect, rendering the solutions quantitatively or even qualitatively irrelevant. Some such dangers cannot be avoided, but some can be aided with enough resources available on projects and applications background.

Applications Background

As before, we call for more textbooks and/or online resources adequate to advanced undergraduate level. In addition to projects, such resources would provide applications background (AB) for interdisciplinary projects reaching to a variety of applied science and engineering. AB topics should be described, including enough applications-level details and units while isolating certain fundamental first-order effects to be solved with the basic-level mathematics defined earlier in this chapter. These resources can be used by faculty developing class projects and building coursework. In larger programs, one can build coursework out of AB materials alone.

Various AB topics have been developed in applied mathematics research journals such as *SIAM Journal of Applied Mathematics*, *Mathematical Methods in the Applied Sciences*, *Mathematical Models and Methods in Applied Sciences*, *Mathematical Modelling and Simulation*, and *SIAM Review*; but these topics are not easily available at the advanced undergraduate level appropriate for mathematics majors. A notable exception is relatively good availability of recently published monographs and texts on mathematical biology, an area where the readers easily connect their own intuition and experiences with the subject.

The subjects developed in disciplines other than life sciences are typically further from intuition, and they come with their own baggage of nomenclature, jargon, and physical principles. Since many mathematics students have only a rudimentary knowledge of physical sciences (acquired perhaps at a level before calculus), the entry barrier to such projects is high, and the expertise required to understand and contribute to new mathematical models comes with a price: time commitment. The situation is perhaps easier for mathematics faculty involved in mathematical modeling, who may already have experience with some applied mathematical modeling or interdisciplinary research; but the challenges of time investments when working outside their own area remain.

We list a few specific examples of AB topics that are needed to develop projects of current interest, leading students to possible future involvement in material science, energy and natural resources, climate and sustainability, and biotechnology. These topics should be explained on the basis of ODEs, LA, SP, and MV, while reaching to advanced mathematics, and in particular to fundamentals of partial differential equations and stochastic processes.

Basic principles of mechanics of fluids and solids helps to understand glaciers, tsunamis, earthquakes, avalanches, and land slides. The flow and transport processes coupled to chemical and biochemical reactions with ties to thermodynamics and across various spatial and temporal scales need to be laid out to understand dangers and management of groundwater contamination and nuclear stockpiles. As an example, the concepts of diffusion can be easily explained without teaching Fourier analysis, Green functions, or Sobolev spaces. Mechanical and electrical systems such as extensions to the balls and springs and networks in Strang [1986] and Strogatz [2001] can be covered to promote interest in alternative energy resources. One can connect music theory to Fourier analysis as was done in Moler [2011]. Network science is pretty well developed and intuitive while remaining highly relevant. Discrete models of particles and individuals such as in pedestrian flow models and their continuous limits are very attractive, giving quick intuitions on how to proceed. Students who master these AB topics can proceed easily to the higher-grade AMM of today.

Modeling Error and Validation/Verification

When teaching AMM, perhaps the hardest thing is to teach students to fail and recover, since mathematics is expected to neatly tie up all the loose ends. Exact solutions to LA problems and separable ODEs are really attractive to (some) students because they come with recipes. But facing reality and messy coupled nonlinear models, students fall apart, and for a good reason. Thus, we should always teach students workaround solutions, and the ability to isolate what can and what cannot be done with a given simple model.

Mathematical modeling is complex; and every model and its solution represent work in progress, while the answers are only approximate. Hence a temptation to formulate templates, a popular remedy to those who "just want to learn." However, once students master the templates, it can be disappointing and frustrating when they cannot apply them again. Thus, we should acknowledge that there is always a modeling error, and we must "un-teach" the misguided concept that at the end of the road there is a neatly-packaged final answer with integer solutions. It helps to recall that the celebrated diffusion and heat equation models are not accurate, since they are based on the idea of infinite speed of propagation. However, they are useful and apply reasonably well to a large set of real-life situations while remaining classic mathematical topics.

The "incomplete" and "messy" character of modeling can be emphasized at every step, and students should be encouraged to question the models given to them, and simultaneously should be encouraged to learn the methods to solve the simplified models. At the heart of, or at least at the completion of, every project, a student should confront the reality, such as the use of physical units as well as of real data and real-life coefficients. After all, without this experience, we may never reach Mars!

The following example illustrates the confusing aspect of units. When integrating a continuous and bounded function on interval (a, b), we are told that a good approximation is obtained with Riemann sums if the size of the subinterval $h \to 0$. Since students are told that the approximation error converges to 0 as $O(h)$, they understand how to evaluate it when $a = 0,\ b = 1$, because h is a fraction less than 1. But in some choice of units, b could just as well be 10^6, and a reasonably-sized h could certainly be greater than 1. Do we still have convergence of the approximation error to 0?

Program Development

In an applied and computational mathematics curriculum, there are many opportunities to include mathematical modeling in basic and advanced coursework as discussed earlier, in individual projects, and in other activities. When building a new program or option, or revising existing ones, one should include all these if possible.

In every program, one should, to the extent possible, include a variety of courses and leave plenty of space for choices. In an AMM curriculum, one should include enough solid (and new) mathematics rather than only use mathematical methods developed at lower levels. Furthermore, the curriculum of applied mathematics needs to be revised frequently so as to stay relevant in view of rapid progress in science and technology. While inclusion of classic subjects and building foundations is an important element of every stable program, one should critically and frequently revisit the details and reconsider emphasis placed on any individual component. This applies to AMM and / or AB coursework and in particular to case studies, so that we avoid leading students into obsolete albeit well understood mathematical islands.

Applied mathematics programs frequently allow for, or require, capstone or senior projects, and promote undergraduate research. Extensions of many AMM coursework projects can be a starting point for capstone, senior thesis, or research projects. To move beyond classroom experience, these require either a further in-depth study of applied mathematics methods and background such as those listed in AB, and / or expertise in computational techniques through coursework in numerical analysis and scientific computing. Since "applied mathematics is just like non-applied, except harder," this requires substantial commitment from faculty and students so as not to change the challenging and useful nature of applied mathematical modeling into something resembling "Mathematics for Poets" or "Poetry for Mathematicians."

Because of the various challenges that we listed above, development of courses and programs in interdisciplinary mathematical modeling takes time. Since enthusiasm may be difficult to sustain long term, one should consider starting small, to the extent possible. When developing projects, consider them to be multi-layered and multi-level so they can be satisfying and motivating for everybody. When assigning grades, try using a "derivative of learning" rather than any absolutes as the basis. Allow for redoing projects. After all, students learn by doing, especially with regard to unit conversion; and applied mathematical modeling does not have only "right" or "wrong" answers.

Computational Skills

It is the 21st century, and technology is ubiquitous in life and education. In mathematical modeling coursework, one finds quickly that it is hard to obtain solutions to the projects without computational technology. Even solving a linear system or diagonalizing a matrix of substantial dimension can be a hurdle, since real-life data rarely give eigenvalues that are conveniently integers. Thus, various tools should be considered in AMM projects, ranging from scripting languages and environments such as MATLAB or Python, to even the use of spreadsheets. There is of course a difference between *using* tools and *developing* them (programming), but one can blend these two and help students progress from one to another.

Mathematics students are not always geeks, but they want to learn enough technology to be successful in the workplace and/or future education. In our experience, mathematical modeling projects provide good motivation for students to use the technology for simple tasks such as plotting, and to learn a bit of programming. However, requiring too much can be overwhelming and detract from the essence of AMM coursework, which should be the mathematical models and methods. We believe in gradual steeping up the challenges and providing enough motivation so that students learn by doing. Simple programming concepts, such as using variables, loops, functions, conditional statements, etc., can be taught "on the side" of any mathematical modeling course. Thus instructors may want to maintain their own and develop their student's proficiency in the use of technology.

Mathematical Modeling in Graduate Education

Most of the above discussion pertains to undergraduates and gradual building of expertise and interests. Things are different for graduate students starting in a new program; they can be the toughest customers of AMM courses. At best, they know (some) models and (some) methods. At worst, they know neither, and thus have to build from basic undergraduate level in ODEs, LA, SP in spite of having achieved graduate-level expertise in other non-applied coursework. This situation can be disappointing and discouraging for those with large ambitions before they even have a chance to discover the challenges up to their intellectual standards. Thus, it seems that the best strategy for graduate students in the "never-ever" category is, unlike with undergraduates, to offer depth rather than breadth, and fully immerse them in one (or at most in very few) challenging mathematical applications, such as those cataloged in Fowler [1997]. Students with (some) experience may enjoy either depth or breadth, or (hopefully) both.

Summary and Conclusions

We have presented a collection of thoughts on program and course development in interdisciplinary mathematical modeling. We hope that these notes spur some interest and help those who are thinking of (re-)developing interdisciplinary mathematical modeling curricula and programs. In general, we find that more resources, institutional investments, and injection of dynamic current projects are needed, and our hope is that the readers will do that. Our work has been inspired and supported by many colleagues from Mathematics Dept. at Oregon State University, and the true judges have been and will be our students, who continue to provide the inspiration.

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