4 Key Problems for Course

1. A beam of spin 1/2 particles is sent through a series of three Stern-Gerlach (SG) measuring devices. The first SG device is aligned along the z-axis and transmits particles with $S_z = \hbar/2$ and blocks particles with $S_z = -\hbar/2$. The second device is aligned along the **n** direction and transmits particles with $S_n = \hbar/2$ and blocks particles with $S_n = -\hbar/2$, where the direction **n** makes an angle θ in the *x*-*z* plane with respect to the *z* axis. Thus particles after passage through this second device are in the state $|+\rangle_n = \cos(\theta/2)|+\rangle + \sin(\theta/2)|-\rangle$. A third SG device is aligned along the z-axis and transmits particles with $S_z = -\hbar/2$ and blocks particles with $S_z = \hbar/2$.

   a) What fraction of the particles transmitted through the first SG device will survive the third measurement?

   b) How must the angle θ of the second SG device be oriented so as to maximize the number of particles that are transmitted by the final SG device? What fraction of the particles survive the third measurement for this value of θ?

   c) What fraction of the particles survive the last measurement if the second SG device is simply removed from the experiment?

2. Consider a three dimensional ket space. In the basis defined by three orthogonal kets $|1\rangle$, $|2\rangle$, and $|3\rangle$ the operators *A* and *B* are represented by

$$A \doteq \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \qquad B \doteq \begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix},$$

   where all the quantities are real.

   a) Do the operators *A* and *B* commute?

   b) Find the eigenvalues and eigenvectors of both operators.

   c) Assume the system is initially in the state $|2\rangle$. Then the observable corresponding to the operator *B* is measured. What are the possible results of this measurement and the probabilities of each result? After this measurement, the observable corresponding to the operator *A* is measured. What are the possible results of this measurement and the probabilities of each result?

   d) How are questions (a) and (c) above related?

3. A beam of identical neutral particles with spin 1/2 travels along the *y*-axis. The beam passes through a series of two Stern-Gerlach spin analyzing magnets, each of which is designed to analyze the spin projection along the *z*-axis. The first Stern-Gerlach analyzer only allows particles with spin **up** (along the *z*-axis) to pass through. The second Stern-Gerlach analyzer only allows particles with spin **down** (along the *z*-axis) to pass through. The particles travel at speed v_0 between the two analyzers, which are separated by a region of length ℓ_0 in which there is a uniform magnetic field B_0 pointing in the *x*-direction. Determine the smallest value of ℓ_0 such that only 25% of the particles transmitted by the first analyzer are transmitted by the second analyzer.

4. Let the matrix representation of the Hamiltonian of a three-state system be

$$H \doteq \begin{pmatrix} E_0 & 0 & A \\ 0 & E_1 & 0 \\ A & 0 & E_0 \end{pmatrix}$$

using the basis states $|1\rangle$, $|2\rangle$, and $|3\rangle$.

a) If the state of the system at time $t = 0$ is $|\psi(0)\rangle = |2\rangle$, what is the probability that the system is in state $|2\rangle$ at time *t*?

b) If the state of the system at time $t = 0$ is $|\psi(0)\rangle = |3\rangle$, what is the probability that the system is in state $|3\rangle$ at time *t*?