RLC Circuits and the Concept of Resonance

Concept

The purpose is to explore the behavior of RLC circuits based upon series and parallel LC combinations. Analyses will be performed in both the time domain (response to an impulse input signal) and in the frequency domain (response to a single frequency sine wave). The use of RLC circuits as passive bandpass and notch filters will be investigated. Fourier analysis of a complicated waveform will be introduced.

Helpful hints and warnings

The "ground symbol" in a circuit implies that the grounds (outer conductors or shields) of the signal generator and the oscilloscope are connected to the circuit at this point. Unlike the DMM, the signal generator and oscilloscopes grounds can be connected only to the circuit ground. Thus, in the bandpass RLC circuit, the oscilloscope can be used only to measure the potential across the parallel LC combination. In the notch-filter RLC circuit, the oscilloscope can be used only to measure the potential across the series LC combination.

To read the inductance on the encapsulated inductors, look for three numbers such as 151. The first two digits are the real first two digits of the inductance. The third digit is the order of magnitude or power of ten. So, 151 means an inductance of 150 something. To figure out what "something" is, the type and size of the inductor needs to be considered. For the encapsulated inductors in the laboratory, the unit is microHenry or μH. So, 150 μH = 0.15 mH. Measure both the inductance and resistance of your inductor using the LRC meter.

For measurements, be sure to vary the frequency of the applied signal over a wide range, such as 100 Hz to 1MHz, to make sure that you are working in the right range for your choice of R, L and C.

Measure values of all components used in your circuits. L and C values can be measured using the laboratory RLC meter.

Assigned Problems

1. From Simpson

Problems 2.12, 2.23

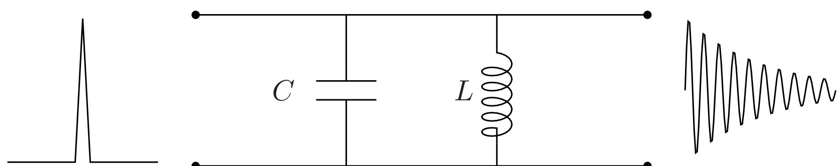
Experimental Instructions

2. Parallel LC resonant circuit

The parallel LC resonant circuit pictured below has a resonant frequency $\omega_\circ = 1/\sqrt{LC}$. The inductor is not ideal in the sense that it has a finite internal resistance R_L which can be modeled as a resistor in series with the inductor.

a. Time-domain analysis: response to a current impulse input signal.

In this picture, a very short current impulse applied to the inductor will induce an oscillation. This is analogous to hitting a pendulum at rest or ringing a bell. The resistance of the inductor and/or the input resistance of the oscilloscope will lead to a decay of the signal. You will need to build an impulse source that is not exactly the ideal current impulse but useful nonetheless. A better source would be an inductively-coupled current, that is a transformer structure that is beyond our capability in the laboratory.



Unit Current Impulse

$V(t) = V_\circ e^{-t/\tau} sin(\omega t)$

(a) A periodic impulse signal requires a function generator which has a highly variable duty factor. Since the function generators cannot generate an impulse (about 1 μsec) at a 1 kHz rate, you must build such a source. The TA will guide you in this effort.

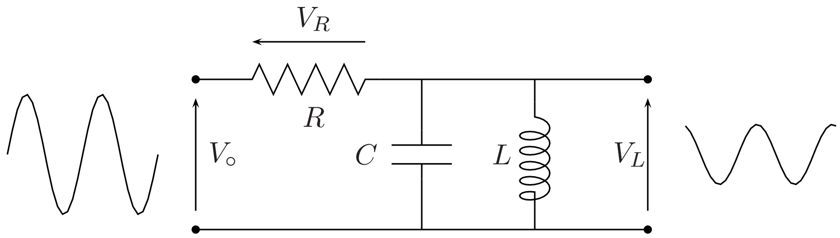
(b) Construct the parallel LC circuit above, using values of L and C such that the frequency $\nu = \omega/2\pi$ is about 50 kHz. Measure the resistance r of the inductor before you place it in the circuit. Add a resistor in series with the $L||C$ part of the circuit, known as the *LC tank*, with $R \approx 200\Omega$.

(c) Apply the impulse signal, record the response and explain the behavior of this circuit to the impulse.

b. Time-domain analysis: response to a square wave input.

(a) Apply a relatively low frequency square wave to this circuit and measure the signal across the LC tank. Sketch the waveform and determine the frequency of any oscillation that is present. Scan the frequency of the square wave from 100 Hz to about $\nu/10$ and determine if there is any frequency dependence to the appearance of the signal across the LC tank.

c. Frequency-domain analysis: response to a sine-wave input.



(a) Apply a sine wave to the circuit and measure the transmission function $|A| = |V_L/V_\circ|$ and the phase difference as a function of the frequency ω. Scan the frequency from 50 to 10^6 Hz. Be sure to take at least 20 data points in the vicinity of the resonant frequency and about 20 points over the rest of the range.

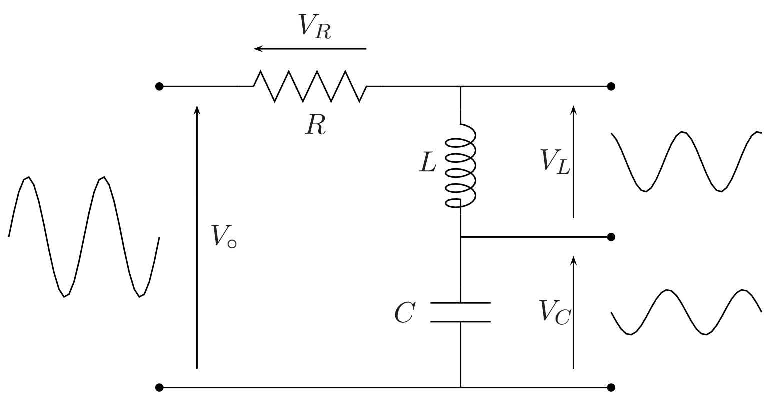
(b) Graph $|A|$ in dB and the phase difference as a function of $\log \nu$, and compare these graphs with theoretical expressions.

(c) Measure the *full-width half-maximum* (FWHM) bandwidth, denoted as $\Delta\omega$. Determine Q from your data and compare it to the theoretical value.

3. Series LC circuit

The series LC resonant circuit pictured below has a resonant frequency $\omega_\circ = 1/\sqrt{LC}$.

a. Frequency-domain analysis: response to a single frequency sine wave input.



(a) Construct this circuit, using values of L and C such that the frequency $\nu = \omega/2\pi$ lies in the 50 to 200 kHz range. Measure the resistance of the inductor before you place it in the circuit. Use $R \approx 1k\Omega$.

(b) Apply a sine wave to the circuit and measure the signal across the inductor and the capacitor, that is, $V_L + V_C$. Notice that it is also possible to measure just V_C with the oscilloscope. Record the transmission function $|A| = |V_{out}/V_{in}|$ and the phase difference as a function of the frequency ω. Scan the frequency from 50 to 10^6 Hz. Be sure to take at least 20 data points in the vicinity of the resonant frequency.

(c) Graph $|A|$ in dB and the phase difference as a function of $\log \nu$, and compare these graphs with theoretical expressions.

(d) Determine Q from your data and compare it to the theoretical values.

(e) From the theoretical analysis of this circuit, what is the current I as ω varies from 0 to ∞?

4. Parallel LC resonant circuit as a Fourier analyzer

A square wave can be described as a sum of sine waves of different frequencies. In fact, a square wave at frequency $\omega_\circ$ can be approximated by a finite sum over sine waves at frequencies $\omega_\circ, 3\omega_\circ, 5\omega_\circ, \ldots$. The parallel LC circuit can be used to determine the relative contributions of the different frequency components.

The laboratory components are not sufficient to perform a true experimental Fourier analysis, which requires variable capacitors and inductors in order to tune LC tanks to these specific frequencies. However, it is possible to perform a qualitative Fourier analysis by using a fixed-frequency LC tank and determining the frequencies for which there are local peaks in the transmission function $A(\omega)$. Suppose you build a circuit such that $\nu_\circ = \omega_\circ/2\pi = 1$ MHz. You should find that, for a square wave input at 500 kHz, $A(500\text{ kHz}) = 0$ because that square wave cannot contain a sine-wave component at 1 MHz. Nonzero values of $A(\nu)$ should be found only 1.5 MHz, 2.5 MHz, etc., that is, only at odd harmonics. So, when the frequency ν of a square wave is varied from 0 to 1 MHz, $A(\nu)$ will be nonzero only when $(2m+1)\nu = 1$ MHz, for $m = 0, 1, 2, \cdots$.

a. Build a parallel RLC circuit with $\nu_\circ = 1$ MHz. Apply a square wave at exactly the resonant frequency and record $|A|$.

b. Decrease the frequence continuously, noting that A decreases and then reaches another maximum. Record the frequency of this maximum, which should be $\omega_\circ/3$, and A at this frequency. Find as many more maxima as as possible and record A and ω for each.

c. Discuss how your data relates to discrete Fourier analysis of square waves.