Capacitors and Time-Dependent Signals

Concept

The purpose is to learn about time-dependent (AC) analysis of RC circuits using a function generator and an oscilloscope. The transient response of an RC circuit will be studied in the time-domain using first a switch and a DMM and then the combination of a square-wave from a function generator and an oscilloscope. Frequency-domain behavior will be measured as well, and the response function of RC circuits will be determined. Complex impedance will be introduced, and Fourier analysis of complicated waveforms will be presented.

Helpful hints and warnings

The "ground symbol" in a circuit implies that the grounds (outer conductors or shields) of the signal generator and the oscilloscope are connected to the circuit at this point. Unlike the DMM, the signal generator and oscilloscopes grounds can be connected only to the circuit ground. Thus, in the low-pass RC circuit, the oscilloscope can be used only to measure the potential across the capacitor. Conversely, in the high-pass CR circuit, the scope can be used only to measure the potential across the resistor.

To read the capacitance on the brown plastic capacitors, look for three numbers such as 153. The first two digits are the real first two digits of the capacitance. The third digit is the order of magnitude or power of ten. So, 153 means a capacitance of 15,000 something. To figure out what "something" is, the type and size of the capacitor needs to be considered. In this case, the unit is picoFarad or pF. So, 15,000 pF = 15 nF = 0.015 μF. As with resistors, do not rely on the code for an accurate value of C. Always measure the capacitance using the *LRC meter*, which can also measure the inductance L.

Three types of capacitors are use in this laboratory. *Ceramic* capacitors exhibit a low capacitance/volume value, a high maximum potential difference rating and low *inductance*, which makes them suitable for high frequency applications. *Polymer* or *plastic* capacitors have a higher C/volume value, a lower maximum potential difference rating and a higher inductance, making them suitable for medium frequency applications. *Electrolytic* capacitors have a large C/volume value, a low working maximum potential difference and slow time response. Furthermore, electrolytics are *polarized*, that is, one side must always be positive with respect to the other to avoid electrochemistry, heating and component failure. As an example of the effect of improper orientation in a circuit, if you expect a potential of 10 V across the capacitor, you will measure only 8 V and power will be dissipated in the device.

Be sure to vary the frequency of the applied signal over a wide range, such as 100 Hz to 1MHz, to make sure that you are working in the right range. You will need to measure the frequency ν each time you change it. Do so by measuring the *period* T as precisely as possible and using $\nu = 1/T$. The horizontal time scale on the oscilloscope is valid only when the *calibrate knob* is properly positioned. Remember the 2π! The *angular frequency* ω used in expressions is $\omega = 2\pi\nu$.

Since the *return* or ground line of the oscilloscope is connected to earth ground, it is possible to observe the time-dependent potential difference $V(t)$ only between a point in the circuit and earth ground. You cannot measure $V(t)$ across an individual resistor or capacitor unless one side is connected to earth ground.

Assigned Problems

1. From Simpson

Problems 2.4, 2.8, 2.10, 2.14, 2.22.

Experimental Instructions

1. Capacitors, function generators and oscilloscopes

a. Listen to the instructor describe different types of capacitors.

b. Follow the instructions of the instructor and familiarize yourself with all aspects of the function generator and oscilloscope.

2. Time-dependent analysis of RC circuits

a. Manual switch model for charge-discharge cycles:

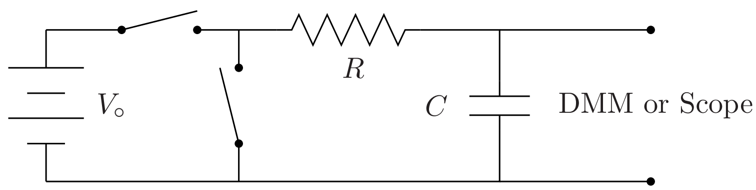

Figure 1: The theoretical model uses switches to charge and discharge a capacitor.

(a) To charge the capacitor, open the switch to ground before making the connection to the ideal potential source. To discharge the capacitor, open or *break* the connection to the power supply before *making* or closing the connection to ground.

(b) The object is to determine the "$1/e$" rise and fall time (τ, a *characteristic time*) of the potential across the capacitor V_c for the particular RC combination. Ideally, $\tau = RC$ and $V_c(t) = V_\circ e^{-t/\tau}$ for discharging and $V_c(t) = V_\circ(1 - e^{-t/\tau})$ for charging.

(c) One issue is how long to wait for the charge or discharge to be complete.

(d) Another issue is the input resistance R_{in} of the instrument measuring V_c. This load resistance on the RC circuit has an effect on both the ultimate potential of the fully-charged capacitor and τ.

(e) A real battery or power supply is not an ideal potential source. The output resistance R_{out} of the supply will effect the maximum value of V_c and the measured τ. The open-circuit potential of the power supply is $V_\circ$.

(f) Cables connecting an RC circuit to the power supply and measuring instrument have capacitance C_s and C_m, respectively. Furthermore, the supply and measuring instrument have capacitances C_{out} and C_{in}.

(g) Write complete expressions for $V_c(t)$ for both charging and discharging in terms of $V_\circ$, R_{out}, R_{in}, C_{out}, C_s, C_m, C_{in}, R and C.

b. Experiment sing the square wave signal from the function generator instead of mechanical switches:

(a) Issues to consider: One difference between using the function generator and the mechanical switches is the fact that the output resistance of function generator $R_\circ$ effects both the charging and discharging dynamics.

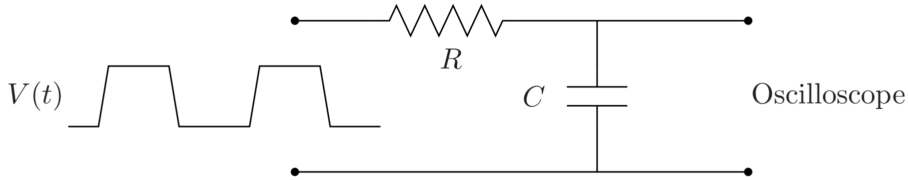

Figure 2: Use a unipolar (no negative values) square wave as input. Other resistances and capacitances might contribute noticeably to the behavior of the circuit.

(b) Read the manuals, and record the output resistance and capacitance for the function generator and the input resistance and capcitance for the oscilloscope. Measure or otherwise determine the capacitances of the cables.

(c) Choose values of R and C such that $\tau = RC \approxeq 0.0001$ second. Assure that τ is not noticeably effected by other experimental capacitances and resistances. Use a carbon film resistor and a mylar capacitor.

(d) Run the initialization program for the function generator and the oscilloscope. Set the function generator to supply a 100 Hz square wave with no offset. Connect the output of the function generator to channel 1 of the oscilloscope and to the RC circuit. Connect the output of the circuit to channel 2. Connect the TTL output (a 5 V square wave) to the external trigger input of the scope. On the trigger menu set the trigger source to external and the trigger level to + 0.7 V. This will provide a reliable trigger regardless of the amplitude of the signals on channels 1 and 2. Set the oscilloscope to display one cycle and to utilize the full vertical range of the display.

(e) Use the waveform acquisition program to acquire channels 1 and 2 simultaneously. The graphs that are automatically generated can be saved as pdf files. The program will ask for the name of a cvs file in which to save the waveforms. The plotting program can be used to read the cvs file and plot the two waveforms.

(f) Measure τ by determining the time for the output to drop to $1/e$ of the maximum and to rise to $1 - 1/e$ of the maximum. Are these two values of τ equal? Does either equal the product RC? Explain any differences you observe.

(g) Vary the frequency from 0 to 100 kHz and determine the frequency range over which the behavior of the circuit is similar to the 100 Hz behavior.

3. The RC Integrator

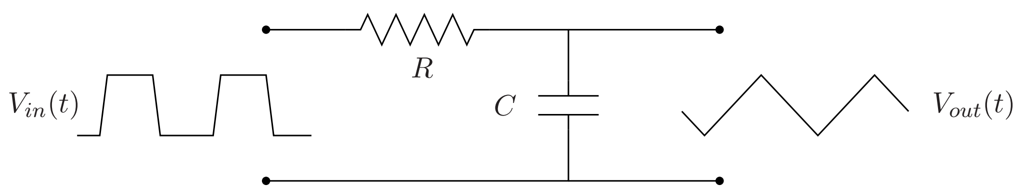

Figure 3: At a sufficiently high frequency, the RC circuit becomes an integrator.

a. Use the circuit of Part 2b and apply a 100 kHz square-wave signal.

b. Explain how the observed waveform is consistent with the concept of an RC circuit behaving as an integrator.

c. Over what frequency range does the circuit behave as an integrator, that is, capable of producing a triangle-wave output from a square-wave input?

d. Try a triangle wave input at 100 kHz and explain the observed waveform.

4. The CR Configuration

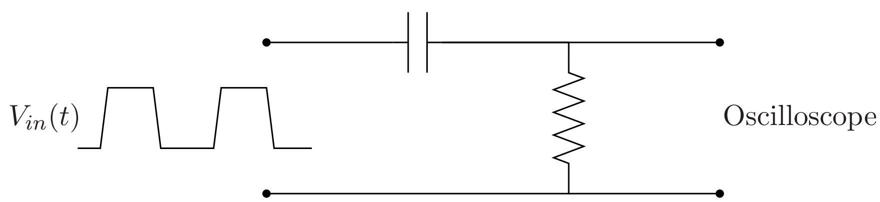

Figure 4: Use a unipolar (no negative values) square wave as input.

a. Reverse the positions of R and C in the circuit of Part 2b.

b. Vary the frequency from 0 to 100 kHz and describe the behavior of this circuit. Does it ever appear to behave as an integrator or a differentiator?

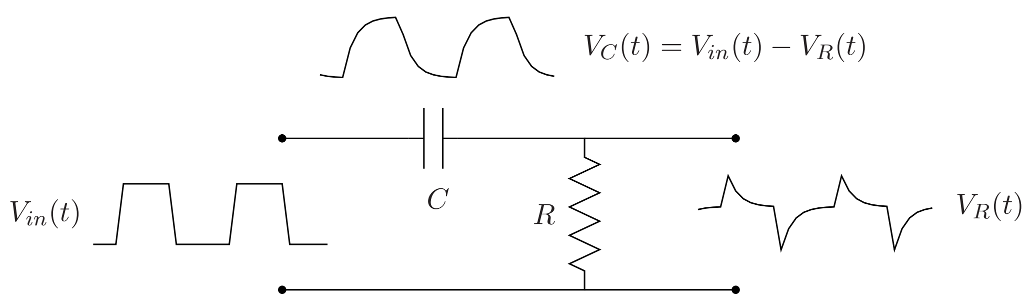

Figure 5: Use a unipolar (no negative values) square wave as input.

c. Try a triangle-wave input at 100 Hz and explain the observed waveform.

5. Response of both configurations to complex waveforms

a. The concept is to apply a visually complex waveform with three frequency components and observe the effects of transmission through RC and CR configurations. Begin with $V_\circ(t) = \cos\omega_\circ t$. Modulation with the function $A(t) = \frac{1}{2}(1+\cos\omega_m t)$ yields

$$V(t) = A(t)V_\circ(t) = \frac{1}{2}\cos\omega_\circ t + \frac{1}{2}\cos\omega_m t\cos\omega_\circ t = \frac{1}{2}\cos\omega_\circ t + \frac{1}{4}\left[\cos(\omega_\circ+\omega_m)t + \cos(\omega_\circ-\omega_m)t\right] \ .$$

The Fourier transform of $V(t)$ will yield a spectrum $v(\omega)$ with only three frequency components at $\omega_\circ$ and $\omega_\circ \pm \omega_m$. The power spectrum is proportional to $|v(\omega)|^2$, and, when presented as $20\log|v(\omega)|$ in dB, it will exhibit peaks at $\omega_\circ \pm \omega_m$ which are smaller than the peak at $\omega_\circ$ by 6 dB.

b. Set the function generator to produce a sine wave at 10 kHz, 10 V_{pp} with no offset. Push the modulation button and set the modulation to AM (amplitude modulation), 9 kHz and 100% depth of modulation. Observe the complex waveform on channel 1 versus time. Adjust the time scale so that you can observe two cycles of the lowest frequency modulation. Press the FFT button on either the on-screen menu or the math mode menu. Adjust the horizontal scale to cleary see all three frequency components. Verify that they are the expected frequencies with the expected amplitudes.

c. Apply the modulated signal to the RC configuration, and capture the input and output waveforms as functions of time with the program. Also, use the fft option in the program to generate plots of the input and output power spectra. Save files with the csv extension so that graphs can be generated later using the plotting program. Interpret the appearance of the output waveform versus the input waveform in the time domain and the differences in the power spectra in the frequency domain.

d. Apply the modulated signal to the CR configuration, and perform the same analysis.

e. Draw a conclusion about the behavior or functionality of each configuration.

6. Frequency response of both configurations

a. Construct a low-pass filter with $R = 48$ kΩ and $C = 1$ nF or an equivalent value of RC. Give some thought to possible variation in the load impedance for the function generator. Set the function generator to supply a sine wave with no offset. Connect the output of the function generator to channel 1 of the oscilloscope and to the low-pass filter. Connect the output of the filter to channel 2. Connect the TTL output (a 5 V square wave) to the external trigger input of the scope. On the trigger menu set the trigger source to external and the trigger level to + 0.7 V. This will provide a reliable trigger regardless of the amplitude of the signals on channels 1 and 2. Manually vary the frequency from 10 Hz to 25 MHz and observe the variation in amplitude and phase of the output relative to the input. At 100 kHz, measure both these quantities. Be sure to get the correct sign of the phase.

b. Apply a sine-wave signal to each configuration and sweep the frequency from 0 to 1 MHz. Make at least 20 measurements. Since you will be plotting your data versus the $\log \nu$, make at least two measurements per decade of frequency. Determine the transmission function $A(\omega)$ by dividing the output amplitude by the input amplitude. Be sure to measure the input amplitude from the function generator at each frequency, since the combination of your circuit and the limitations of the generator will lead to a signal that will generally decrease in amplitude with frequency. The applied frequency should be determined by taking the reciprocal of the period measured on the scope. Measure the phase difference between the output and input signals. A good point on the waveform to use for such measurements is the point at which the trace crosses the 0 Volts line. If the period of the input signal is T and the displacement of the output signal is t, then the phase difference is $\phi = 2\pi t/T$.

c. For each configuration, plot the data and theoretical curve together as $20 \log A(\nu)$ vs. $\log \nu$. Determine the *breakpoint* or *characteristic* frequency from the data plot by identifying the -3dB point, and compare this to the theoretical value. Draw conclusions about the behavior of both circuits.

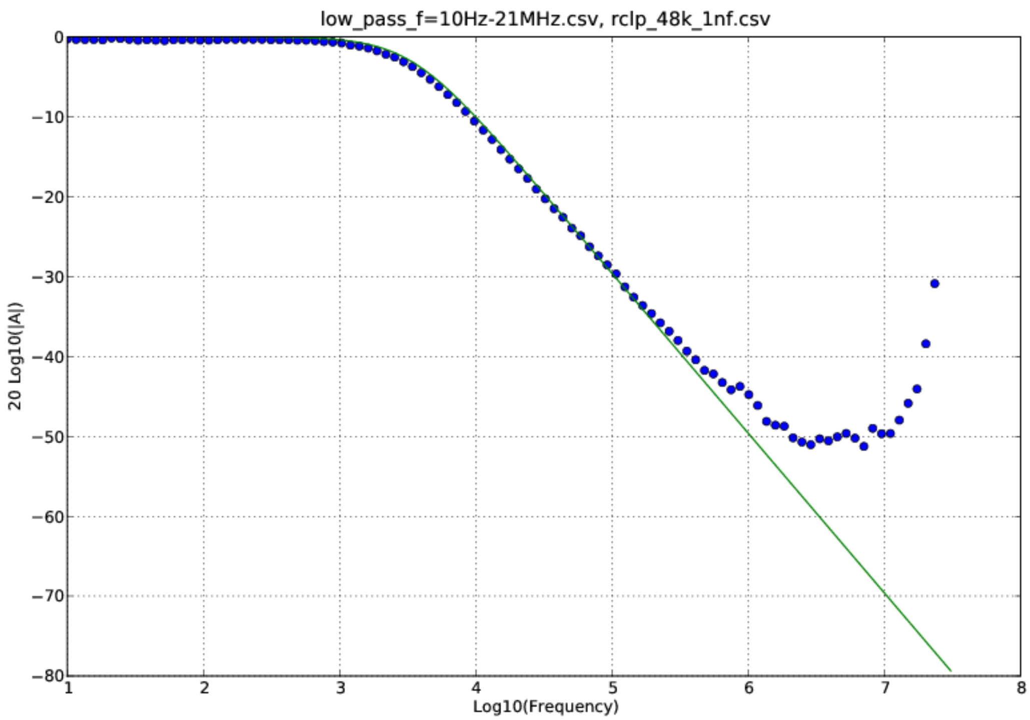

Figure 6: Bode plot for a low-pass RC filter. Indicate the -3 dB or *breakpoint* frequency. The horizontal axis should be ν.