

Introduction to Operational Amplifiers

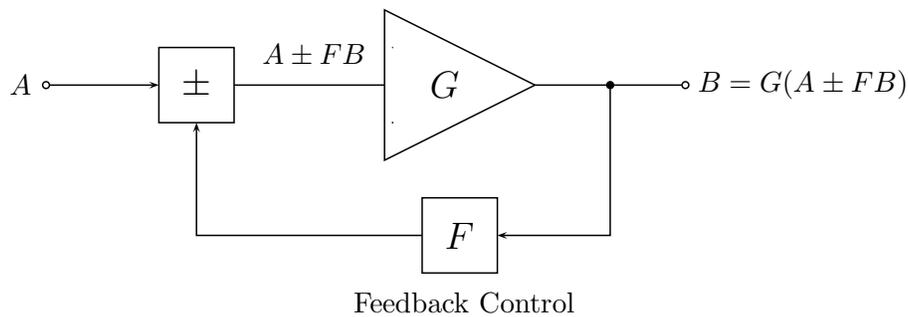
Circuit Functionality

So far, only *passive circuits* (RC , RL and RLC) have been analyzed in terms of the time-domain operator T and the frequency-domain operator $A(\omega)$, and these operators yield output signals that are generally of lesser amplitude than the input signals. The exception, of course, is the LC circuit for which at the resonant frequency ω_0 the amplitude can be larger than the input because the circuit can accumulate energy. In all cases, the output power of a passive circuit is less than the input.

In contrast, an *active circuit* can increase the power of a signal and provide interesting and useful functionality beyond that achievable by a passive circuit. An elementary building block of active circuits is the *ideal amplifier*.

Ideal Amplifier

For our purposes, the ideal amplifier exhibits very large amplitude and power gains at all frequencies. By very large, we mean an amplitude gain on the order of 1 million or more. This seems unrealistic, and indeed it is in terms of power but not in terms of amplitude. Consider the following circuit built from an amplifier with a huge amplitude gain G , a summing device indicated by the \pm symbol and a passive circuit which reduces the amplitude of a signal. G is referred to as the *open-loop gain* of the op amp.



With active circuits there is the opportunity to use *feedback* to provide interesting behavior. Feedback is accomplished by connecting the output back to the input through a circuit with an attenuation factor or feedback fraction F . Given an input signal A , the output signal is $B = G(A \pm FB)$. Thinking in the time-domain, if the “+” operation is used, the output signal will tend toward $\pm\infty$. This is pathological behavior. Thinking about the effect of the “+” operation in the frequency-domain is more interesting, and this will be a subject for later discussion. Considering now only the use of the “-” operation, negative feedback yields a very useful behavior. Solving for the output B in terms of the input A , and using the condition that $FG \gg 1$, we find that

$$B = \frac{G}{1 + FG}A \simeq \frac{G}{FG}A = \frac{A}{F} .$$

This is an incredibly useful result because it states that as long as the inherent gain G of the amplifier is very large and the feedback fraction F is not too small, the gain $1/F$ of the circuit is determined only by

the feedback fraction. Of course, the validity of the approximation is in the eye of the beholder. The first order error can be easily obtained through

$$\frac{G}{1 + FG} = \frac{G}{GF \left(1 + \frac{1}{GF}\right)} \simeq \frac{1}{F} \left(1 - \frac{1}{GF}\right) = \frac{1}{F} - \frac{1}{GF^2} .$$

If $G = 10^6$ and $F = 0.01$, then the circuit gain is

$$\frac{G}{1 + FG} \simeq 100 - .01 = 99.99 ,$$

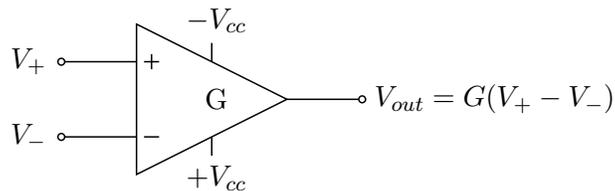
but if $F = 2 \times 10^{-4}$, then the circuit gain is

$$\frac{G}{1 + FG} \simeq 5000 - 25 = 4975 ,$$

a significant error.

The Operational Amplifier

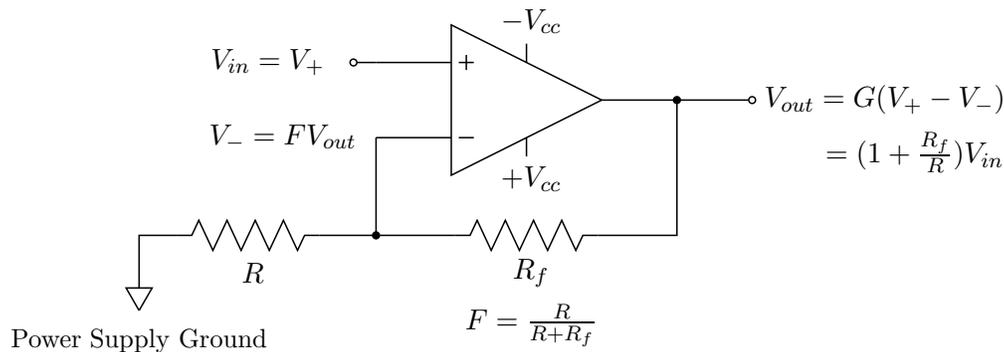
The physical realization of a device with an ideal amplifier and a feedback function is the *operational amplifier* or *op amp*. It is a *difference amplifier*, that is $V_{out} = G(V_+ - V_-)$, with a very large gain.



Furthermore, for the ideal op amp, G is independent of frequency and the input resistances are infinite so that no current flows in or out of the inputs. The only unavoidable reality is the fact that the output potential must lie within the range supplied by the power source, $\pm V_{cc}$. The + and - inputs are referred to as the *noninverting* and *inverting inputs*. Note that if the output is to be about 1 V the difference between V_+ and V_- will have to be $1/G$, that is, only about $1 \mu\text{V}$. An important conclusion is that when the output is not pinned at $\pm V_{cc}$ the inputs are virtually equal.

Non-Inverting Configuration

That this device is equivalent to the conceptual diagram above is shown in this figure, in which negative feedback is achieved by connecting the output to the inverting input through a potential divider with attenuation factor $F = R/(R + R_f)$.



The relationship between the input and output signals for this *noninverting amplifier* configuration is

$$V_{out} = G(V_+ - V_-) = G(V_{in} - FV_{out}) \rightarrow V_{out} = \frac{G}{1 + GF}V_{in} \simeq \frac{V_{in}}{F} = \left(1 + \frac{R_f}{R}\right)V_{in} .$$

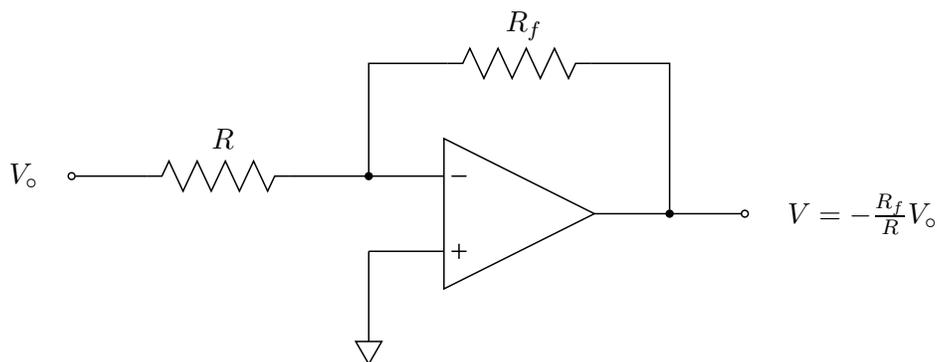
The gain of this circuit is

$$A = \frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R} .$$

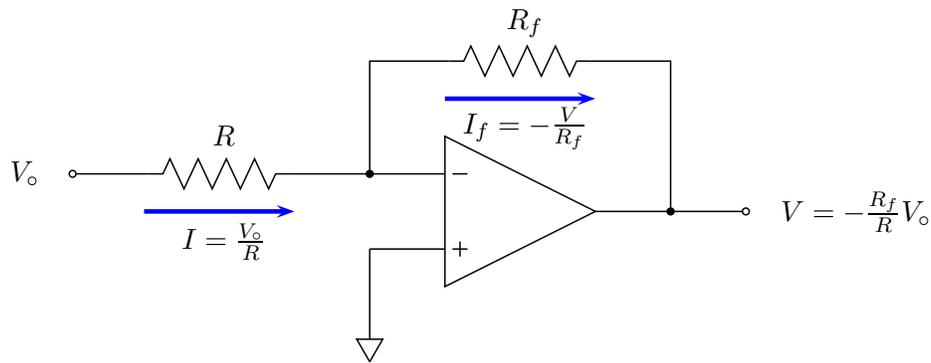
Note also that in this analysis both V_{in} and V_{out} could be functions of time or frequency. Finally, the input resistance of this circuit is the internal resistance of the op amp, which is infinite in the ideal case.

Inverting Configuration

Another way to achieve negative feedback and circuit gain that is determined only by the feedback fraction F is to use the *inverting amplifier* configuration.



If the input signal becomes more positive, then the output should become more negative. However, since this increasingly negative output is fed back to the inverting input the effect is to dampen the change in the output. Another way to view the behavior is to use the two defining properties of an op amp. First, G is huge, so that in order for the output to lie between $\pm V_{cc}$ $V_+ \simeq V_-$. The implication of this statement is that if V_+ is at the system ground, then so is V_- . Hence, when V_+ is grounded, V_- is a *virtual ground*. The object generating the input signal attempts to assert some potential at V_- , but the feed back from the active device holds V_- at ground potential. Second, since no current flows into the op amp, all the current through the input resistor R flows through the feedback resistor R_f to the output of the opamp.



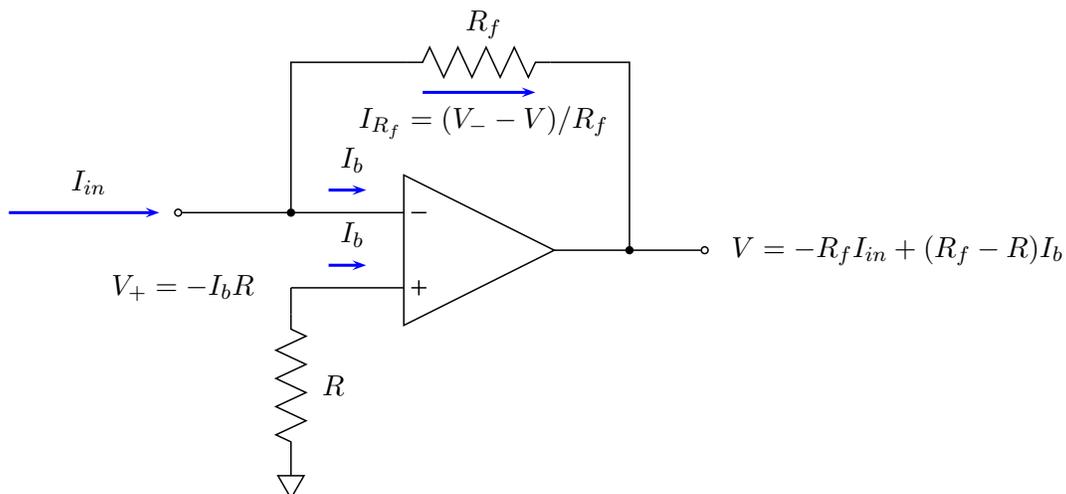
Using the facts that $I = I_f$ and $V_- = V_+ = 0$,

$$I = I_f \rightarrow \frac{V_o - V_-}{R} = \frac{V_- - V}{R_f} \rightarrow \frac{V_o}{R} = -\frac{V}{R_f} \rightarrow V = -\frac{R_f}{R} V_o .$$

This result and that for the gain of the noninverting op amp circuit are independent of frequency because we have presumed that G and all resistances are independent of frequency. Note that the input resistance is simply R because V_- is a virtual ground.

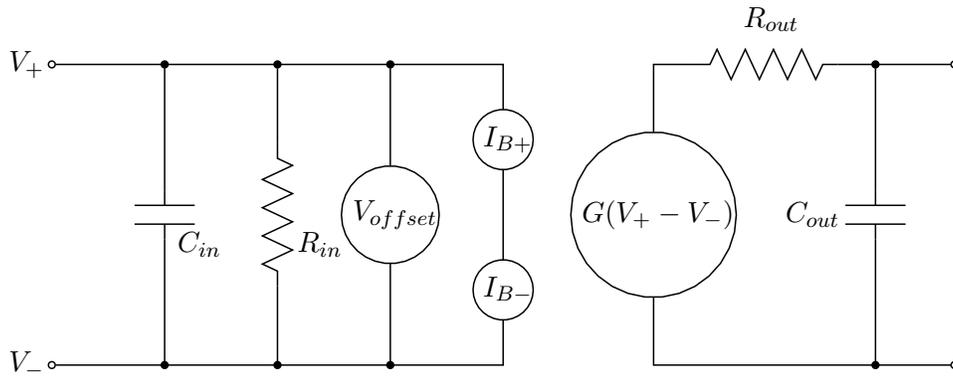
Transimpedance Amplifier

When an ideal current source provides the input signal, an input resistor in the inverting configuration is irrelevant, and the same analysis as performed previously yields the result $V = -R_f I_{in}$ when $R = R_f$. Though R_f is measured in Ohms, the *transimpedance* R_f is usually given in V/A.



Important Non-Idealities of Op Amps

Real op amps are non-ideal in many ways, but it is necessary to consider only the most important non-idealities. This simple model for an op amp includes everything we need to consider.



This is a functional model not a physical model.

Limited Frequency Response

The open-loop gain G of any op amp must decrease with increasing frequency. Some op amps can operate at very high frequencies, while others, such as the simple 741, are of no value above 1 Mhz or so. A reasonable mathematical model for $G(\omega)$ is

$$G(\omega) = \frac{G_o}{1 + i\omega/\omega_o} ,$$

a function reminiscent of the response function of the RC low pass filter,

$$A(\omega) = \frac{1}{1 + i\omega RC} .$$

The quantity ω_o is not a resonant frequency but rather just a parameter for each op amp. The gain of an op amp circuit is then

$$A(\omega) = \frac{G(\omega)}{1 + G(\omega)F} .$$

For low frequency, $G \simeq G_o$ is very large, and

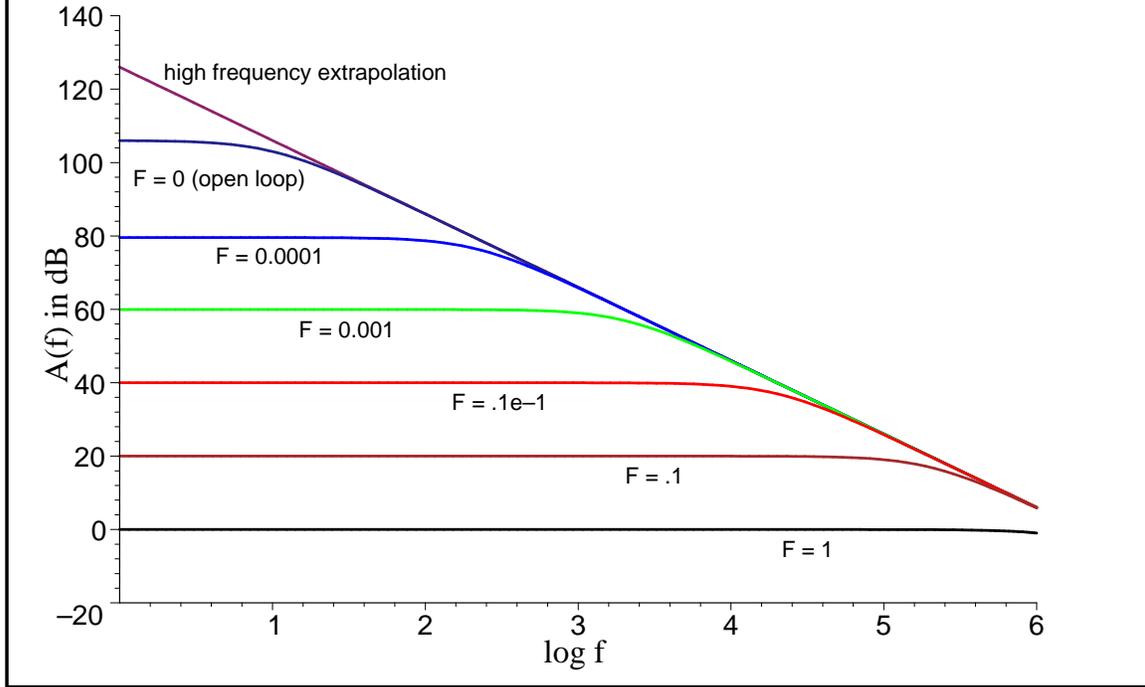
$$A(\omega) \simeq \frac{1}{F} .$$

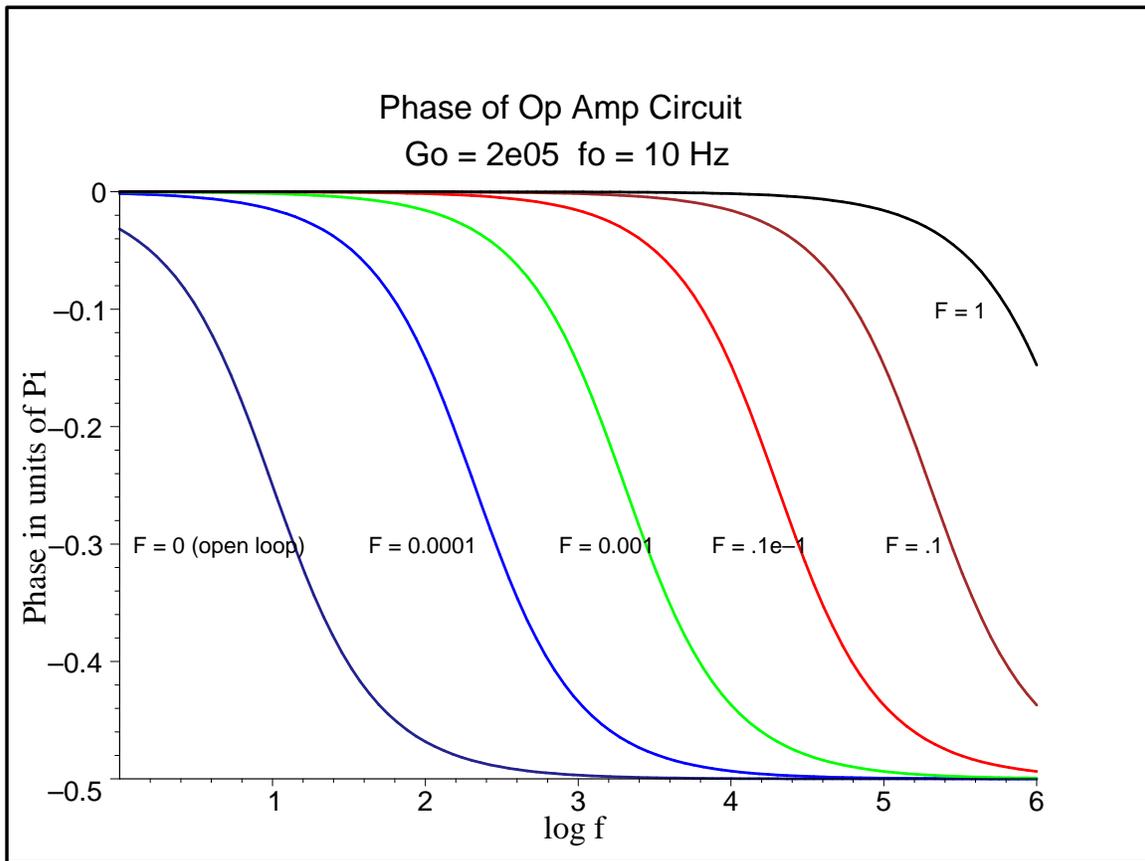
At high frequency, $G \simeq -iG_o\omega_o/\omega$ is much smaller, and

$$A(\omega) \simeq \frac{1}{F + i\omega/\omega_o G_o} \simeq -i \frac{G_o\omega_o}{\omega} .$$

Notice that A is now independent of F . Using typical values for a 741 op amp, $G_o = 2 \times 10^5$ and $\omega_o = 2\pi \times 10$ Hz, the absolute value of A in dB and the phase of A as functions of $\nu = \omega/2\pi$ are

Gain of OpAmp Circuit
 $G_o = 2e05$ $f_o = 10$ Hz





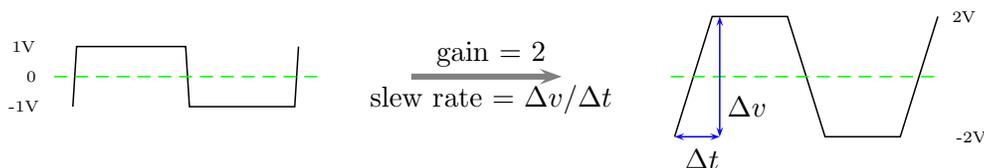
At high frequency,

$$20 \log |A| \simeq 20 \log G_o + 20 \log \omega_o - 20 \log \omega = 20 \log G_o + 20 \log \nu_o - 20 \log \nu ,$$

and the intercept of this straight line at $\nu = 1$ Hz is $20 \log G_o + 20 \log \nu_o$. An important conclusion is that a gain of 1000 can be achieved only for $\nu < 1$ kHz and a gain of 10 can be achieved only up to about 100 kHz.

Slew Rate

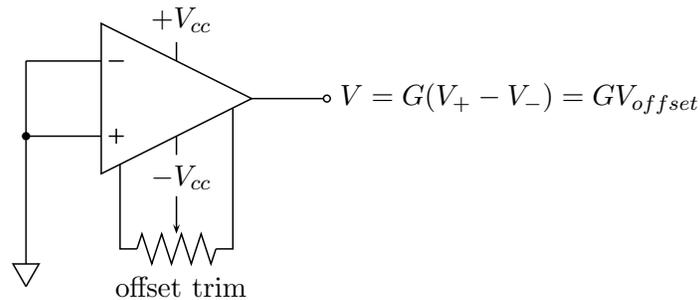
A reality related to the finite frequency response is the *slew rate*, a measure of the maximum rate of change of the output signal. This is typically specified in V/ μ s.



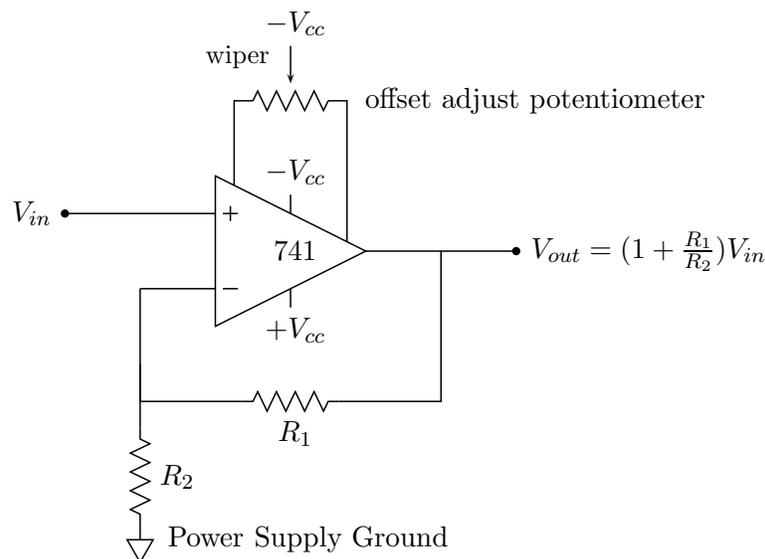
For an ideal square-wave input, the output will exhibit a linear rise and fall dictated by the slew rate. For a sine-wave input, the output could be a triangle or sawtooth waveform when the expected output amplitude is large, but could look more like a sine-wave when the output amplitude is sufficiently small.

Input Offset Potential

An *input offset potential* V_{offset} is always present in an op amp, and it is an internal constant potential difference between the two inputs. The output resulting from this is GV_{offset} when no feedback is present. To correct for this small potential difference, an offset-nulling capability is included inside each op amp. Connecting a trimming potentiometer or *trimpot* to the two offset terminals and applying $-V_{cc}$ allows one to null the output.



If V_{offset} is bothersome, one must ground the inputs of whatever amplifier has been built and adjust the offset-nulling trimpot. For example, in the noninverting configuration, V_{in} must be grounded and the trimpot adjusted to zero the output.



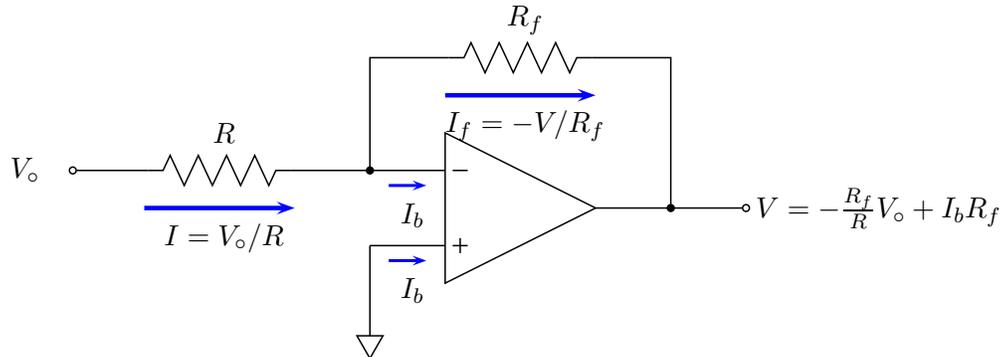
If this correction is not performed, the constant offset potential V_o will produce a constant background,

$$V_{out} = G(V_+ - V_-) = G(V_{in} + V_o - V_-) = G(V_{in} + V_o - FV_{out})$$

$$V_{out} = \frac{G}{1 + GF}(V_{in} + V_o) \simeq \frac{1}{F}(V_{in} + V_o) = \left(1 + \frac{R_f}{R}\right)(V_{in} + V_o) .$$

Input Bias Current

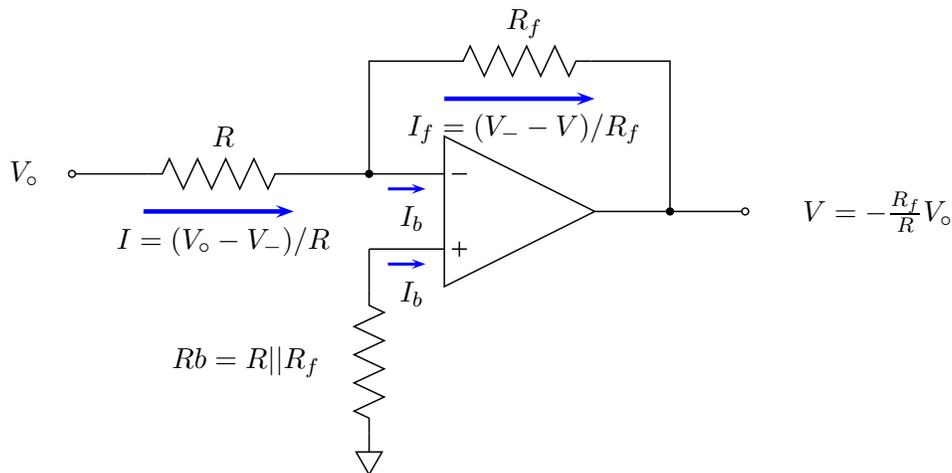
Both inputs to an op amp draw what is called *input bias current*, I_{b+} and I_{b-} . They are generally almost equal, so we will assume only one value I_b for both currents, and that I_b is positive for current flowing into the op amp. For the inverting op amp configuration, the effect of this current is to add a constant to the ideal output.



The analysis is based upon conservation of current, $I = I_b + I_f$, and the fact that $V_- = V_+$:

$$I = I_b + I_f \rightarrow \frac{V_o}{R} = I_b - \frac{V}{R_f} \rightarrow V = -\frac{R_f}{R} V_o + I_b R_f .$$

Since $I_b \simeq 20$ nA for a simple 741 and perhaps as low as 1 femtoamp for a much higher quality op amp, $I_b R_f$ is only a problem when R_f is large. To correct for this error, a resistor R_b can be added to the noninverting input so as to create an input potential $V_+ = -I_b R_b$ which will cancel the additional potential at the inverting input due to I_b .



Again using $I = I_b + I_f$, and $V_- = V_+$,

$$I = I_b + I_f \rightarrow \frac{V_o - V_-}{R} = I_b + \frac{V_- - V}{R_f} \rightarrow \frac{V_o}{R} - V_+ \left(\frac{1}{R} + \frac{1}{R_f} \right) - I_b = -\frac{V}{R_f}$$

$$\frac{V_o}{R} + I_b R_b \left(\frac{1}{R} + \frac{1}{R_f} \right) - I_b = -\frac{V}{R_f} .$$

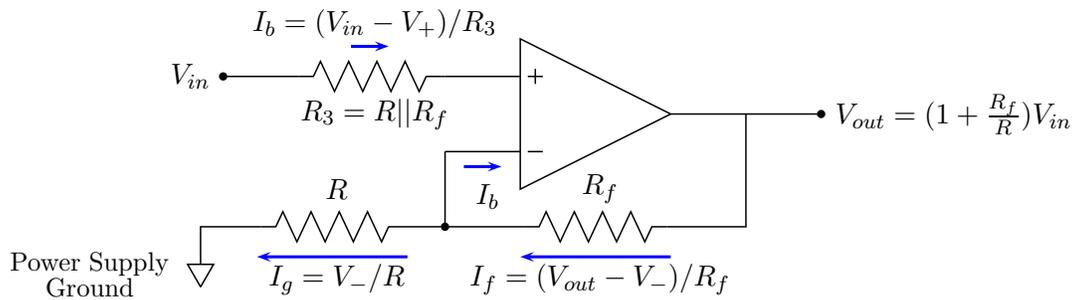
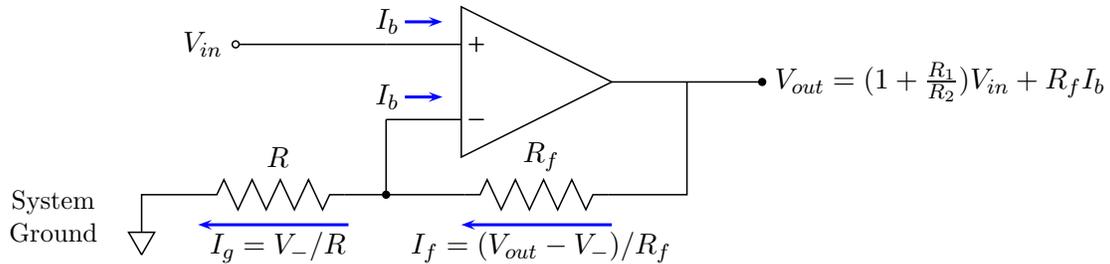
If

$$R_b = \left(\frac{1}{R} + \frac{1}{R_f} \right)^{-1} = R || R_f ,$$

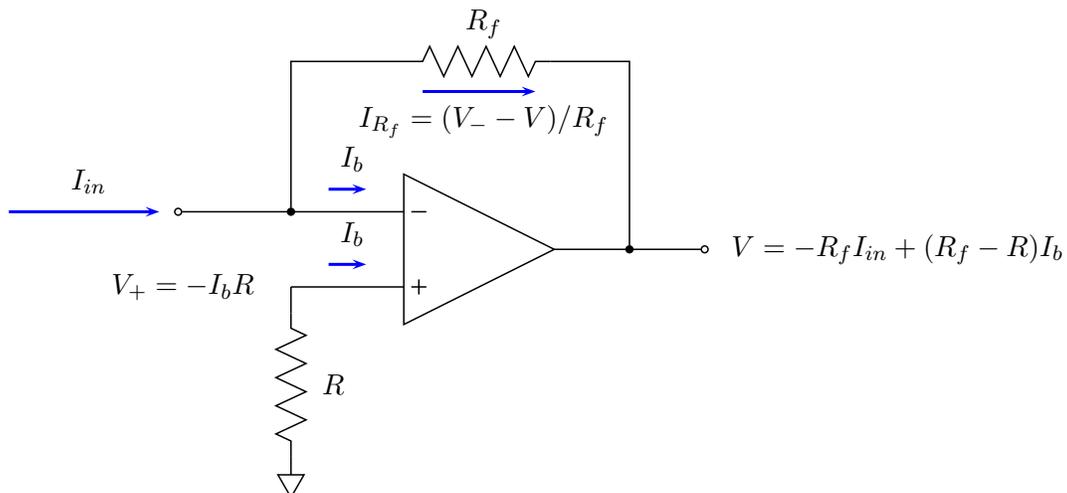
then the I_b term vanishes, and

$$V = -\frac{R_f}{R} V_o .$$

For the noninverting amplifier, the analysis is based upon $V_+ = V_-$, and the result is pictured below.



The transimpedance amplifier can all be corrected for the effect of input bias current as shown here.

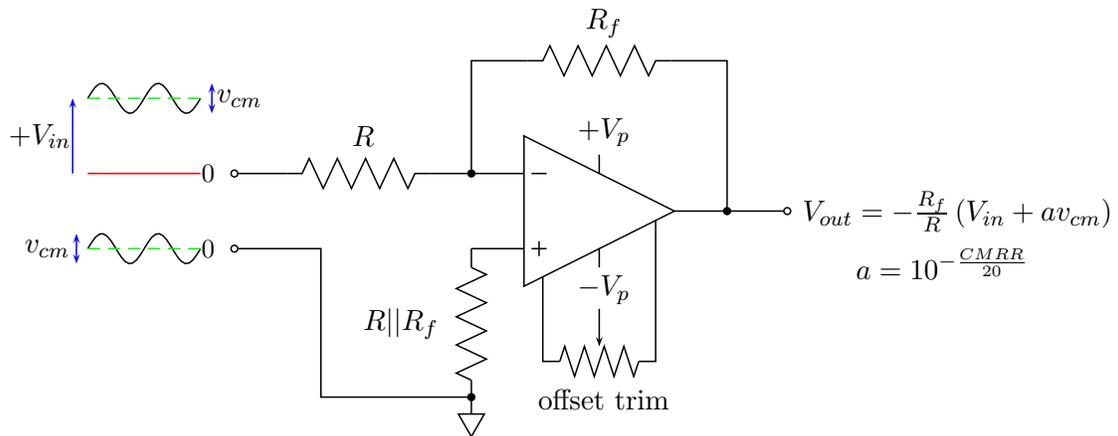
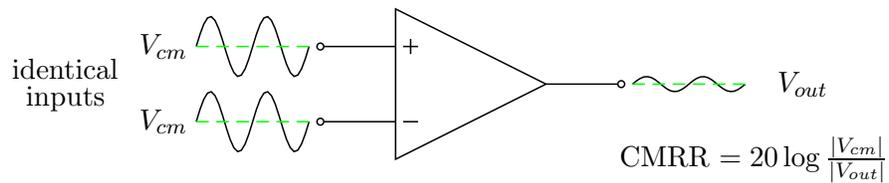


Input Resistance

This input resistance is large but finite.

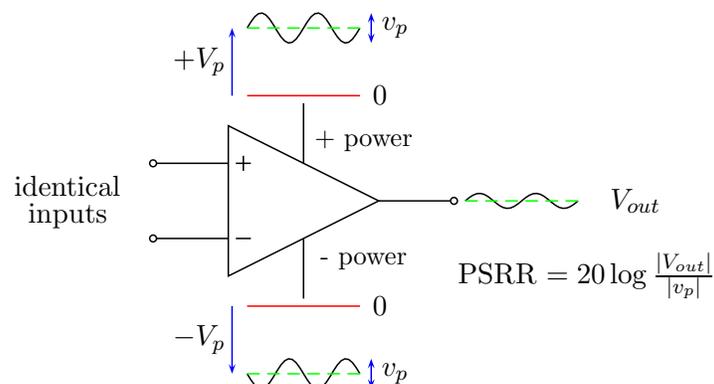
Common Mode Rejection

Common mode rejection ratio CMRR.



Power Supply Variation Rejection

Power supply rejection ratio PSRR.



Physical Op Amp

Dual in-line package (DIP).

