RC Circuits:

With the introduction of the capacitor comes unique electrical response as a function of time.

Time Domain Analysis:

Technique = Kirchoff's energy conservation

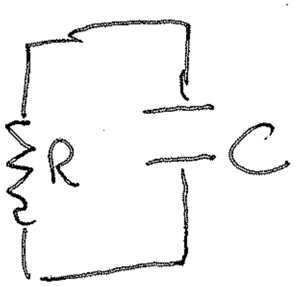
$$\Rightarrow \sum_{\text{closed loop}} \Phi = 0$$

Solve resulting differential equation.

RC Discharging:

Consider a fully charged capacitor (close switch...

$$Q = CV \quad \text{or} \quad C_1 V + C_2 V^2 + \dots$$



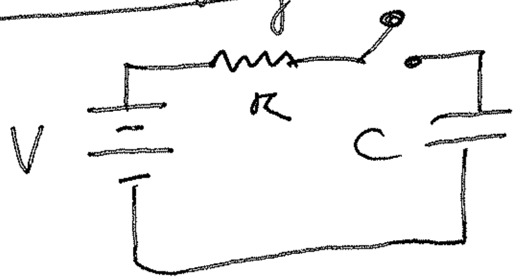
$$V_R = IR, \quad \frac{Q}{C} = V_C$$

$$\Rightarrow \frac{Q}{C} = IR = -R\frac{dQ}{dt} \quad \left(I = -\frac{dQ}{dt}\right)$$

loosing charge

$$\frac{dQ}{dt} = -\frac{1}{RC} Q \Rightarrow \boxed{Q(t) = Q_0 e^{-t/RC}}$$

RC Charging:



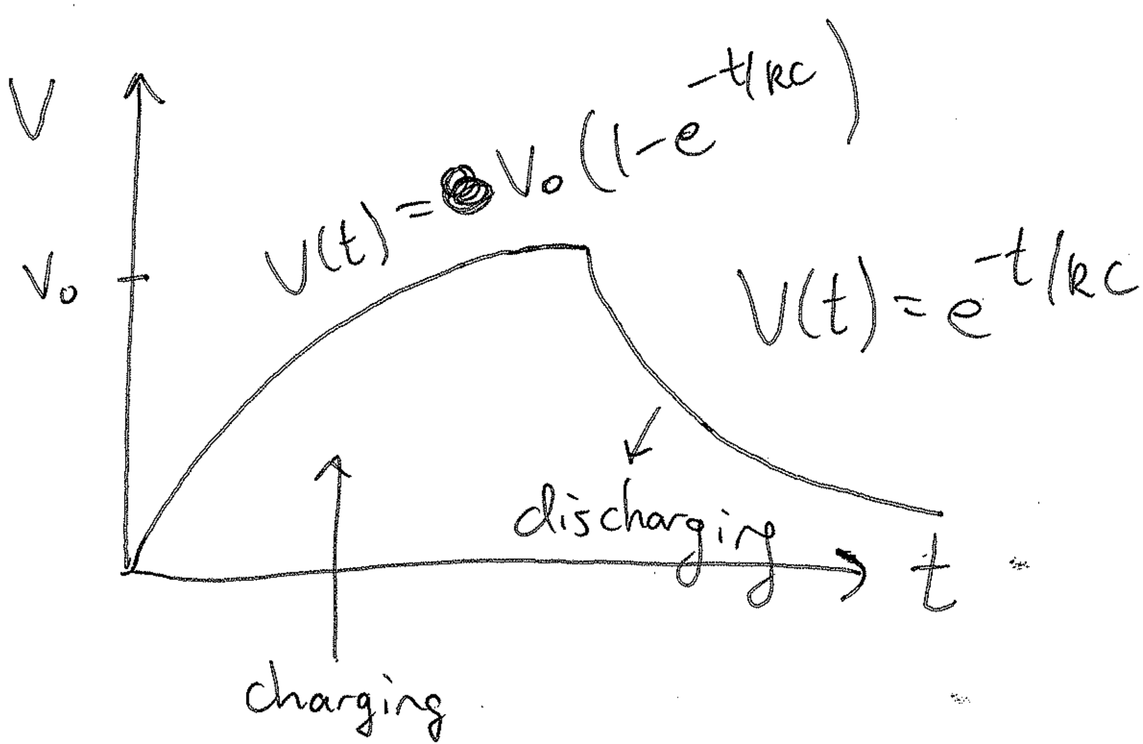
$$\sum \Phi = 0$$

$$V - IR - \frac{Q}{C} = 0$$

$$V - R\frac{dQ}{dt} - \frac{Q}{C} \Rightarrow \frac{dQ}{dt} = \frac{CV - Q}{RC}$$

$$\int_0^Q \frac{dQ'}{Q'-CV} = \int_0^t \frac{-dt}{RC} = -\frac{t}{RC} = \ln(Q'-CV)\Big|_{Q'=0}^{Q'=Q}$$

$$\frac{Q-CV}{-CV} = e^{-t/RC} \quad \text{or} \quad \boxed{Q(t) = CV\left(1-e^{-t/RC}\right)}$$

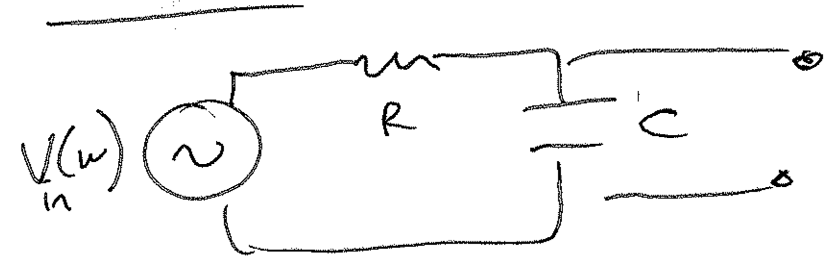


Frequency domain RC :

Now consider some frequency dependent voltage source. Take for example a sin function (ie. $\sin(\omega t)$) instead of a battery. Frequency dependent response is a fundamental physical property.

Capacitor Impedence : $\left(Z_c = \frac{1}{i\omega C} \right)$ Explain ...

RC Filter :

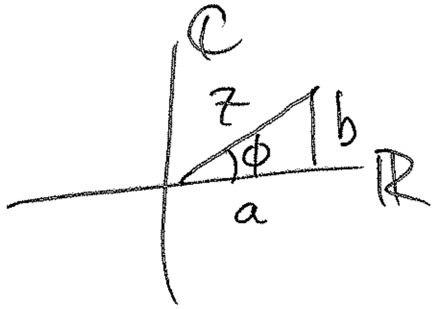


$$V_{out} = \left(\frac{Z_c}{Z_c + R} \right) V_{in}$$

$$\frac{V_{out}}{V_{in}} = A(\omega) = \frac{Z_c}{R + Z_c}$$

$$A(\omega) = \frac{1/i\omega C}{1/i\omega C + R}$$

(Step aside for complex #)



$$z = a + ib$$

$$|z|^2 = a^2 + b^2$$

$$\cos\phi = \frac{a}{\sqrt{a^2+b^2}}, \quad \tan\phi = \frac{b}{a}, \quad \sin\phi = \frac{b}{\sqrt{a^2+b^2}}$$

$$\frac{a+ib}{\sqrt{a^2+b^2}} = \cos\phi + i\sin\phi = e^{i\phi}$$

$$\boxed{a + ib = |z| e^{i\phi}} \qquad \boxed{\phi = \tan^{-1}\left(\frac{b}{a}\right)}$$

Back to 

$$A(\omega) = \frac{\frac{1}{i\omega C}}{\frac{1}{i\omega C} + R} = \frac{1}{1 + i\omega RC}$$

$$\boxed{A(\omega) = \frac{e^{-i\tan^{-1}(\omega RC)}}{\sqrt{1 + (\omega RC)^2}}}$$

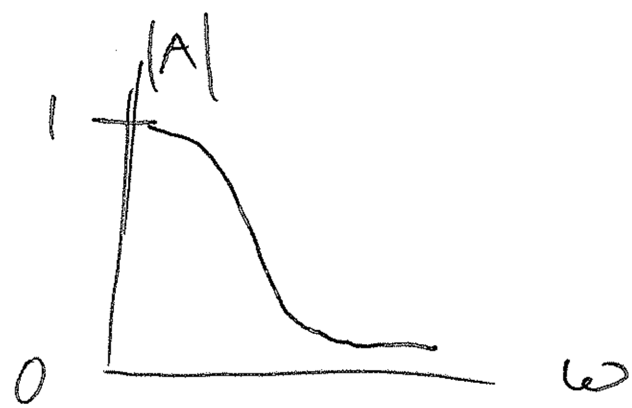
$$\boxed{\text{Phase } \phi = \frac{2\pi \Delta t}{T}}$$

Draw

Now look @ $|A(\omega)|$ and

$\phi(\omega)$ as $\omega \to 0$ and $\omega \to \infty$.

$$|A(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$



$$\phi = -\tan^{-1}(\omega RC)$$